LECTURE
NOTES ON
ELECTRICAL MACHINE
PC EE 401

DEPARTMENT OF ELECTRICAL ENGINEERING

College of Engineering and Management, Kolaghat

Transformers and Generators

Transformers:

The static electrical device which transfers the voltage from one level to another level by the principle of self and mutual induction without change in frequency.

Michael Faraday propounded the principle of electro-magnetic induction in 1831.It states that a voltage appears across the terminals of an electric coil when the flux linked with the same changes. The magnitude of the induced voltage is proportional to the rate of change of the flux linkages. This finding forms the basis for many magneto electric machines

The earliest use of this phenomenon was in the development of induction coils. These coils were used to generate high voltage pulses to ignite the explosive charges in the mines. As the d.c. power system was in use at that time, very little of transformer principle was made use of. In the d.c. supply system the generating station and the load center have to be necessarily close to each other due to the requirement of economic transmission of power.

Transformers can link two or more electric circuits. In its simple form two electric circuits can be linked by a magnetic circuit, one of the electric coils is used for the creation of a time varying magnetic field. The second coil which is made to link this field has a induced voltage in the same. The magnitude of the induced emf is decided by the number of turns used in each coil. Thus the voltage level can be increased or decreased by changing the number of turns. This excitation winding is called a primary and the output winding is called a secondary. As a magnetic medium forms the link between the primary and the secondary windings there is no conductive connection between the two electric circuits. The transformer thus provides an electric isolation between the two circuits. The frequency on the two sides will be the same. As there is no change in the nature of the power, the re suiting machine is called a 'transformer' and not a 'converter'. The electric power at one Voltage/current level is only 'transformed' into electric power, at the same frequency, to another voltage/current level.

Even though most of the large-power transformers can be found in the power systems, the use of the transformers is not limited to the power systems. The use of the principle of transformers is universal. Transformers can be found operating in the frequency range starting from a few hertz going up to several mega hertz. Power ratings vary from a few miliwatts to several hundreds of megawatts. The use of the transformers is so wide spread that it is virtually impossible to think of a large power system without transformers. Demand on electric power generation doubles every decade in a developing country. For every MVA of generation the installed capacity of transformers grows by about 7MVA.

Classification of Transformer:

The transformers are classified according to:

1. The Type of Construction:
   (a) Core Type Transformer

(b) Shell Type Transformer
2. The Number of Phases:
   (a) Single Phase Transformer
   (b) Three Phase Transformer
3. The Placements:
   (a) Indoor Transformer
   (b) Outdoor Transformer
4. The Load:
   (a) Power Transformer
   (b) Distribution Transformer

Ideal Transformer

To understand the working of a transformer it is always instructive, to begin with the concept of an *ideal* transformer with the following properties.

1. Primary and secondary windings have no resistance.

2. All the flux produced by the primary links the secondary winding i.e., there is no leakage flux.

3. Permeability μr of the core is infinitely large. In other words, to establish flux in the core vanishingly small (or zero) current is required.

4. Core loss comprising of *eddy current* and *hysteresis* losses are neglected.

Construction of a Transformer

There are two basic parts of a transformer:

1. Magnetic core
2. Winding or coils

- **MAGNETIC CORE:** The core of a transformer is either square orrectangular in size. It is further divided in two parts. The vertical portion on which the coils are bound is called limb, while the top and bottom horizontal portion is called yoke of the core as shown in

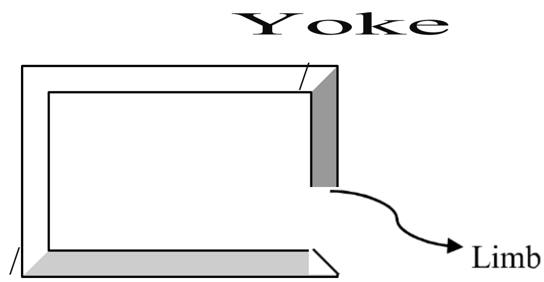


Fig. 2

Core is made up of laminations. Because of laminated type of construction, eddy current losses get minimized. Generally high grade silicon steel laminations (0.3 to 0.5 mm thick) are used. These laminations are insulated from each other by using insulation like varnish. All laminations are varnished. Laminations are overlapped so that to avoid the air gap at the joints. For this generally 'L' shaped or 'I' shaped laminations are used which are shown in the fig. 3 below.

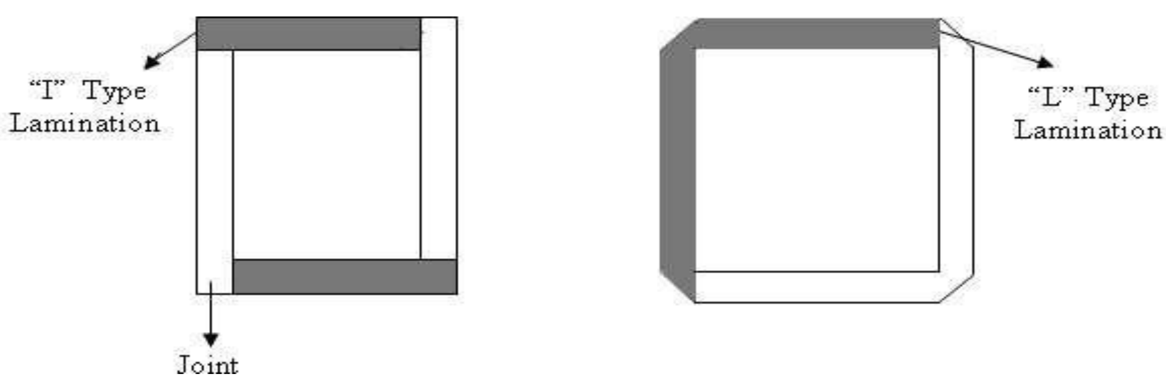
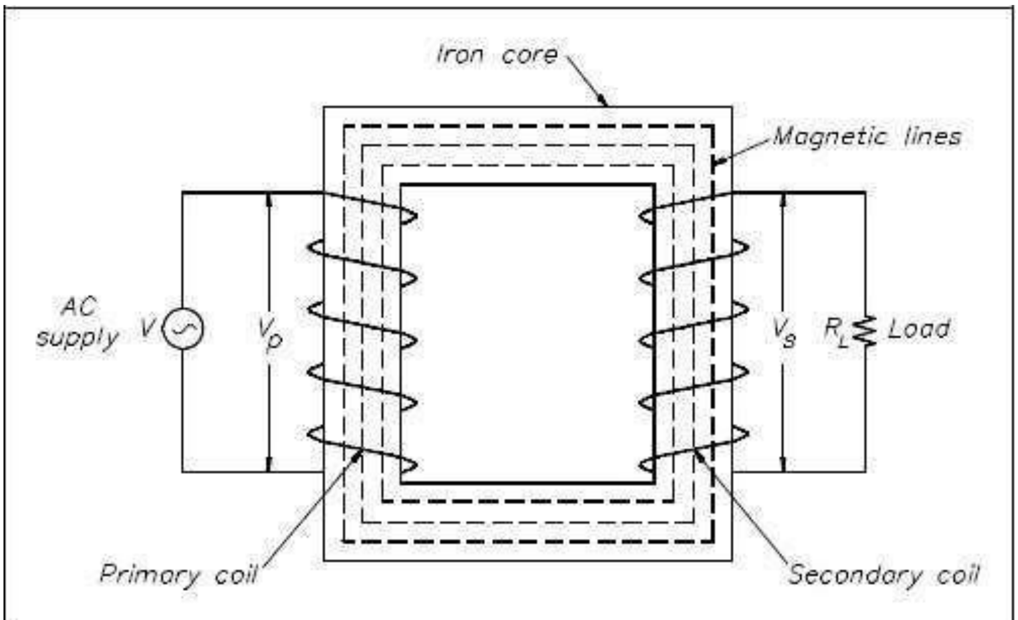


Fig. 3

WINDING: There are two windings, which are wound on the two limbs of the core, which are insulated from each other and from the limbs as shown in fig. 4. The windings are made up of copper, so that, they possess a very small resistance. The winding which is connected to the load is called secondary winding and the winding which is connected to the supply is called primary winding. The primary winding has N_1 number of turns and the secondary windings have N2 number of turns.



N_1 N_2

Fig. 4. Single Phase Transformer

TYPES OF TRANSFORMERS:

The classification of transformer is based on the relative arrangement or disposition of the core and the windings. There are two main types of transformers.

1. Core type
2. Shell type

CORE TYPE:

Fig 5(a)& (b) shows the simplified representation of a core type transformer, where the primary and secondary winding have been shown wound on the opposite sides. However, in actual practise, half the primary and half the secondary windings are situated side by side on each limb,so as to reduce leakage flux as shown in fig 6. This type of core construction is adopted for small rating transformers.

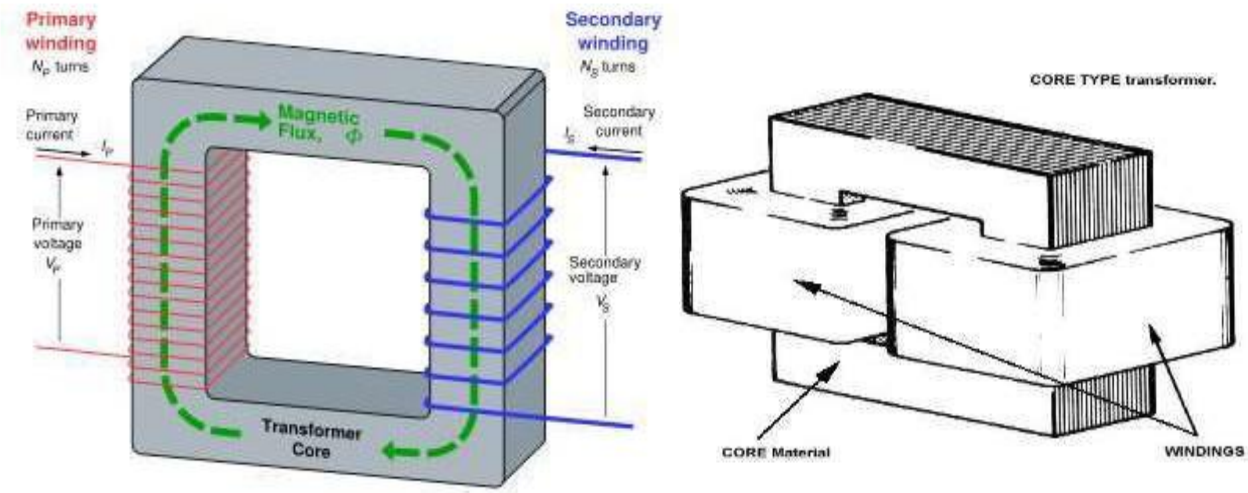
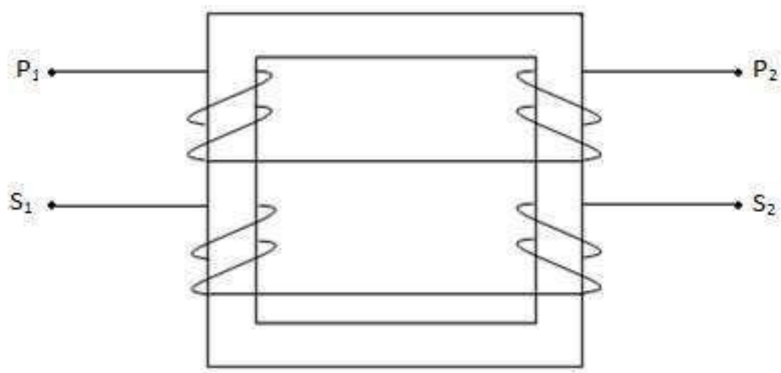


Fig. 5(a) & (b) Single Phase Core Type Transformer



SHELL TYPE:

In this type, the windings occupy a smaller portion of the core as shown in fig 5. The entire flux passes through the central part of the core, but outside of this a central core, it divides half, going in each direction. The coils are form wound, multilayer disc-type, each of the multilayer discs is insulated from the other by using paper. This type of construction is generally preferred for high voltage transformers.

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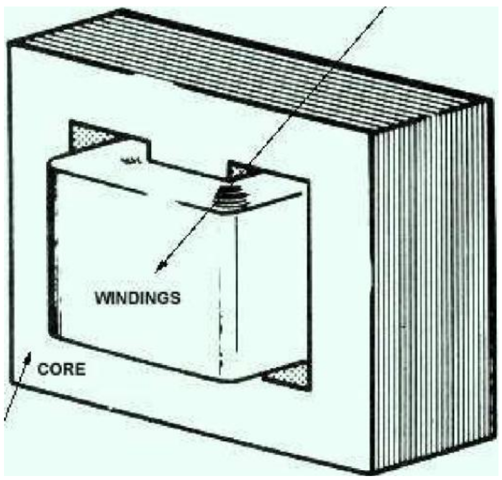
SHELL IYPE tr ansforme,,;



GOR Smaterial made up oftti1n laminate iror, **sheets,** ..ach &h""t is **.coated wilh** an insu lating vam ish **and** the **eralire** core i6 **lh"11 pn,sssd tog..the.-.**

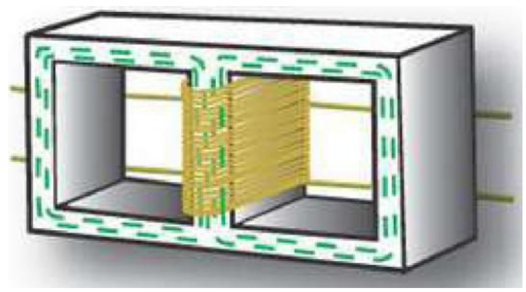
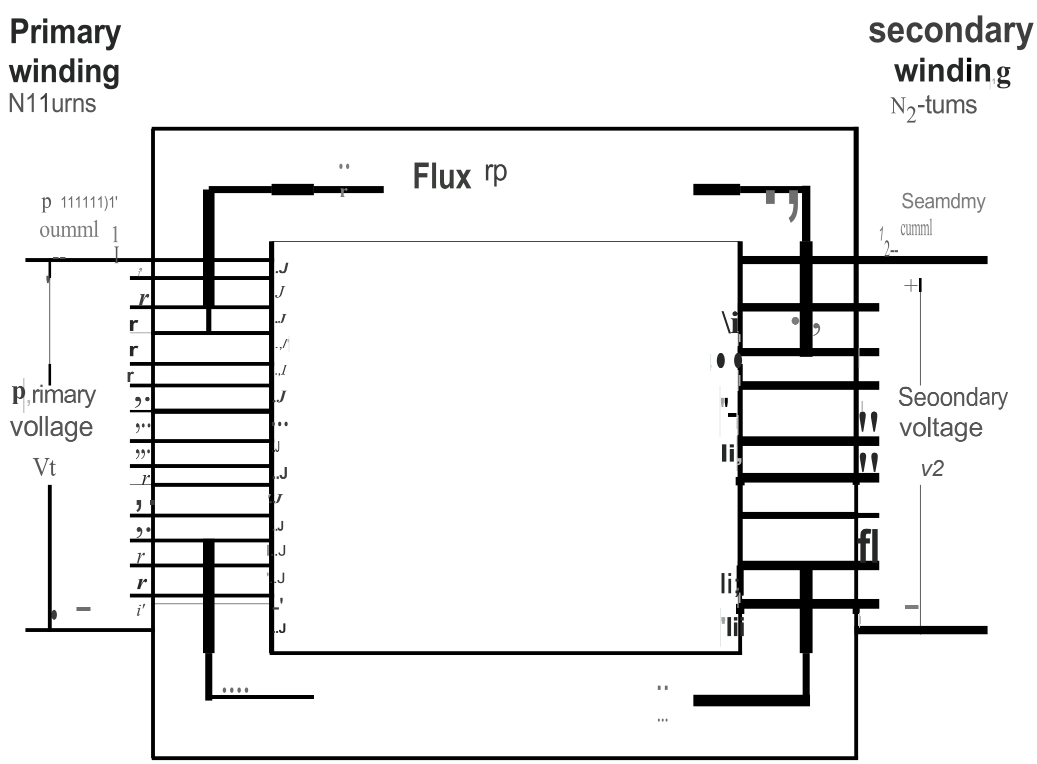


Fig. 7 (a) & (b) Single Phase Shell Type Transforme

Principle of Operation of a Single Phase Transformer

A

single phase



transformer works on the principle of mutual induction between two magnetically coupled coils. When the primary winding is connected to an alternating voltage of r.m.s value, V1 volts, an alternating current flows through the primary winding and setup an alternating flux <!> in the material of the core. This alternating flux cp, links not only the primary windings but also the secondary windings. Therefore, an e.m.f e1 is induced in the primary winding and an e.m.f e2 is induced in the secondary winding, e1 and e2 are given $e1 = -N1\frac{d\varphi}{dt}$ --------- (a)

$e2 = -N2\frac{d\varphi}{dt}$ ------(b)

If the induced e.m.f is e1 and e2 are represented by their rms values E1 and *Ei* respectively, then

$E1 = -N1\frac{d\varphi}{dt}$ ------- (1)

$E2 = -N2\frac{d\varphi}{dt}$ ------- (2)

Therefore, $\frac{E2}{E1} = \frac{N2}{N1} = k$ ---------------- (3)

K is known as the transfonnation ratio of the transformer. When a load is connected to the secondary winding, a current Ii flows through the load, V2 is the terminal voltage across the load. As the power transfered from the primary winding to the secondary winding is same,

Power input to the primary winding = Power output from the secondary winding.

$E_1I_1 = E_2I_2$

(Assuming that the power factor of the primary is equal to the secondary).

Or, $\frac{E2}{E1} = \frac{I1}{I2} = k$ ------- (4)

From eqn (3) and (4), we have

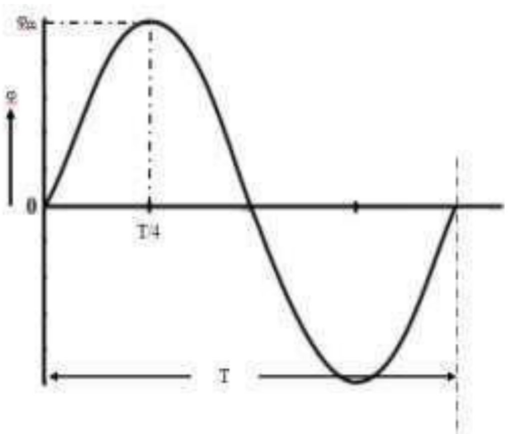
$\frac{E2}{E1} = \frac{N2}{N1} = \frac{I1}{I2} = k$ ----------- (5)

The directions of emfs E1 and *Ei* induced in the primary and secondary windings are such that, they always oppose the primary applied voltage V1.

<u>EMF Equation of a transformer:</u>

Consider a transformer having,

N1 =Primary turns
N2 = Secondary turns
<l>m = Maximum flux in the core
<l>m = Bm x A webers
f= frequency of ac input in hertz (Hz)



The flux in the core will vary sinusoidally as shown in figure, so that it increases from zero to maximum "<l>m" in one quarter of the cycle i.e, $^{1}/_{4f}$ second

Therefore, average rate of change of flux = $^{\phi m}/_{1/4f}$

= 4f<pm

We know that, the rate of change of flux per turn means that the induced emf in volts.

Therefore, average emf induced per turn = 4f<pm volts.

Since the flux is varying sinusoidally, the rms value of induced emf is obtained by multiplying the average value by the form factor .

Therefore, rms value of emf induced per turns= 1.11x4fxcpm

= 4.44f<pm volts

The rms value of induced emfin the entire primary winding = (induced emf per turn) x number of primary turns

i.e, E1 =4.44fq>mXN1 = 4.44IBmxAxN1

Similarly;

Ez= 4.44 f (f)m x Nz = 4.44 f Bm x Ax Nz

Transformation Ratio:

(1) Voltage Transformation Ratio
(2) Current Transformation Ratio

Voltage Transformation Ratio:

Voltage transformation ratio can be defined as the ratio of the secondary voltage to the primary voltage denoted by K

Mathematically given as $K = \frac{Secondary\ Voltage}{PrimarPrimar\ Voltage} = \frac{V_2}{V_1}$

$$K = \frac{E_2}{E_1} = \frac{4.44\ f\ \varphi_m N_2}{4.44\ f\ \varphi_m N_1} = \frac{N_2}{N_1}$$

$$K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1}$$

Current Transformation Ratio:

Consider an ideal transformer and we have the input voltampere is equal to output voltampere.

Mathematically, $Input\ Voltampere = Output\ Voltampere$

$$V_1 I_1 = V_2 I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

$$\therefore,\ K = \frac{V_2}{V_1} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{I_1}{I_2}$$

Coupled circuits

- When two coils separated by each other, a change in current in one coil will effect the voltage in another coil by mutual induction
- Self Inductance: A coil capable of inducing an emf in itself by changing current flowing through it, this property of coil is known as self inductance.
- The self induced emf is directly proportional to the rate of change of current.

ea di/dt; e =L di/dt

- Where L=coefficient of self inductance.

Mutual Inductance

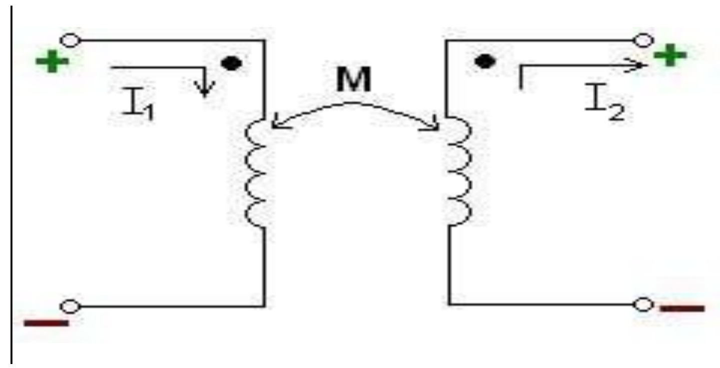
- Current in one coil changes, there occurs a change in flux linking with other as result an emf is induced in the adjacent coils.
- The mutually induced emf e2 in the second coil id dependent on the rate of change of current in the first coil.
- e2 a dil/dt; e2=Mdil/dt

COEFFCIENT OF COUPLING

- K= M/(L1L2)
- The two coils is said to be tightly or perfectly coupled only when K=l and therefore M=L1L2 it's said to be maximum mutual inductance
- When the distance between the two coils is greater than the coils are said to be loosely packed
- Coefficient of coupling will help in deciding whether the coils are closely packed or loosely packed.

Derivation for Co-efficient of coupling

Dot Convention

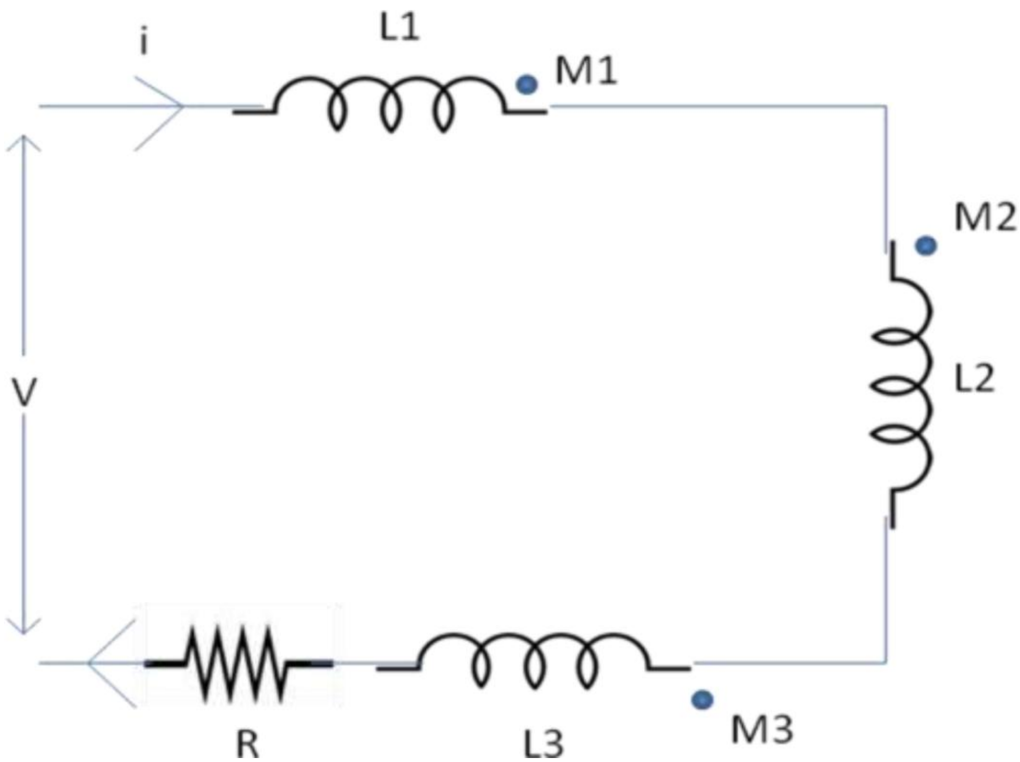


- A current entering the dotted terminal of one coil produces an open-circuit voltage which is positively sensed at the dotted terminal of the second coil
- A current entering the undotted terminal of one coil produces an open-circuit voltage which is positively sensed at the undotted terminal of the second coil.
- The advantage of dot convention is to find out the direction of the winding and direction of flux linking the coil
- The direction of the flux due to rate of change of flux can be analyzed by right hand thumb rule.

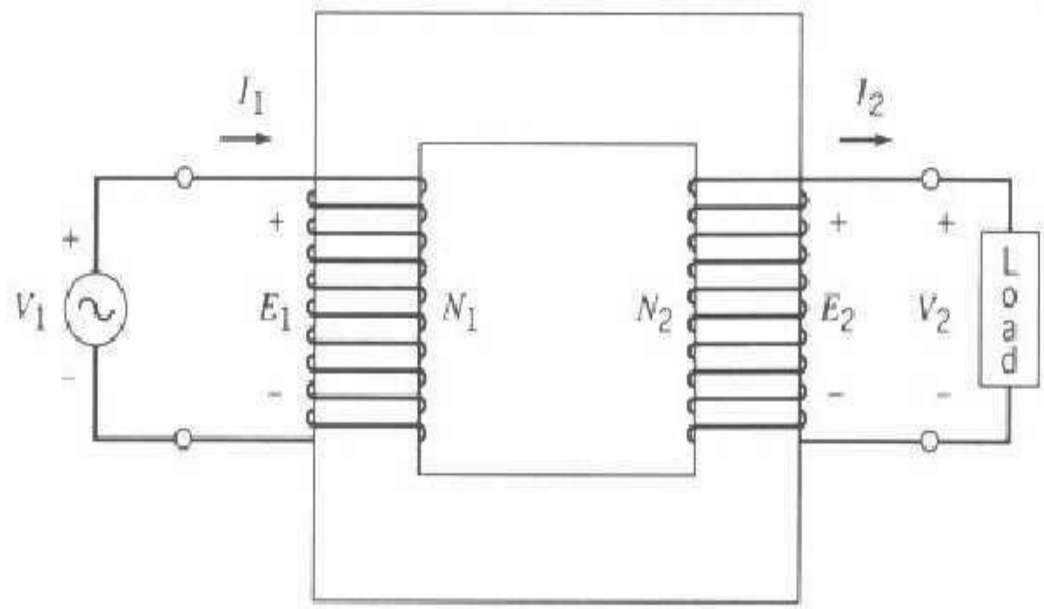
Different connections of coupled circuits

- Series Aiding:
- Series Opposing:
- Parallel Aiding:
- Parallel Opposing
- **Refer Circuit diagram and derivation for the class notes.**

Equilibrium Equations



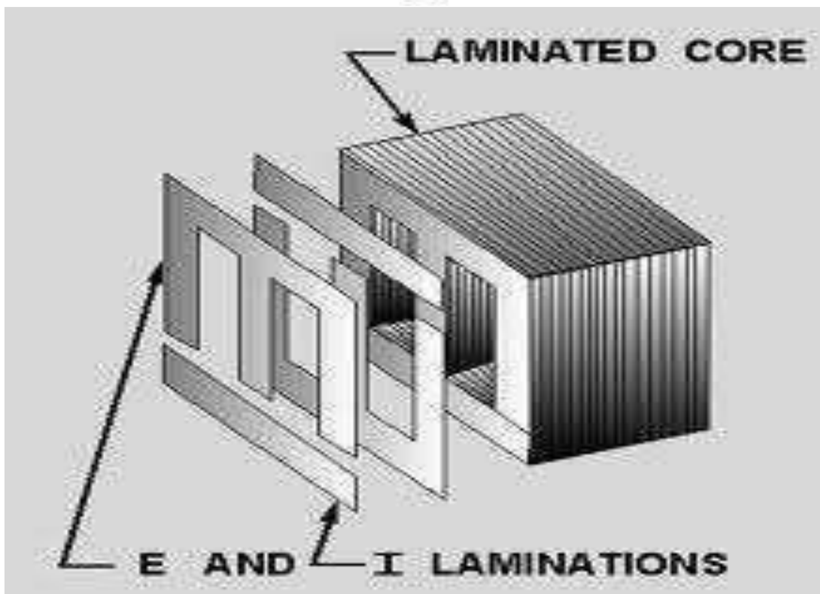
- The coil where electrical energy is fed is considered as Primary
- The coil where load is connected to draw the current from mutual induction 1s Secondary
- There are Two main part in Transformer 1) Core 2) Windings
- Core: The top and bottom part of the core is Yoke, The side limbs are considered as Legs. The core is made up of Silicon steel to avoid the Eddy current and Hystersis Loss.
- Windings: Basically it is made up of Copper and depends on the current value based on this it is of two types Low Voltage and High Voltage Winding.



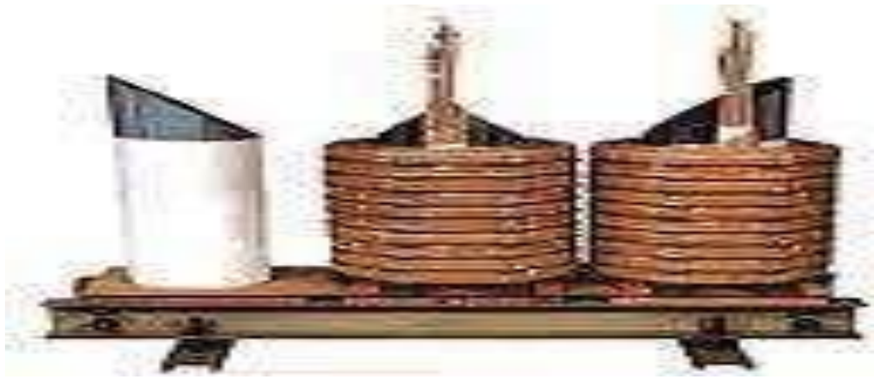
There are Two main part in Transformer

1) Core 2) Windings

- Core: The top and bottom part of the core 1s Yoke, the Vertical portions are considered as of Limbs Legs.
- The core is made up of Silicon steel laminations of thickness 0.33m (CRGO) to avoid the Eddy current and Hysteresis Loss.
- Each laminations are varnished one another and bolted to form a L or T or I shaped structures.



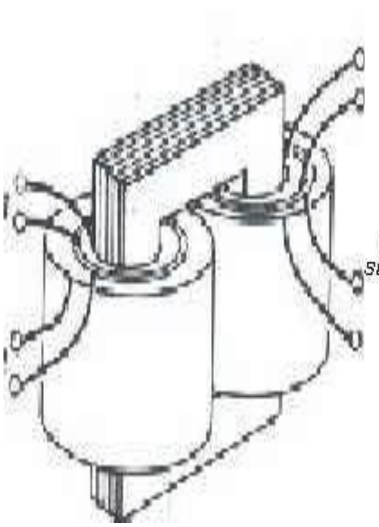
- Windings: Basically it is made up of Copper and depends on the current value based on this it is of two types Low Voltage and High Voltage Winding.
- The LV and HV coils should be placed close to each other as to increase the mutual induction.
- The two coils are separated by insulated materials such as paper, cloth or mica
- Coils maybe placed Helically(Cylindrical) or Sandwiched in the window of transformer

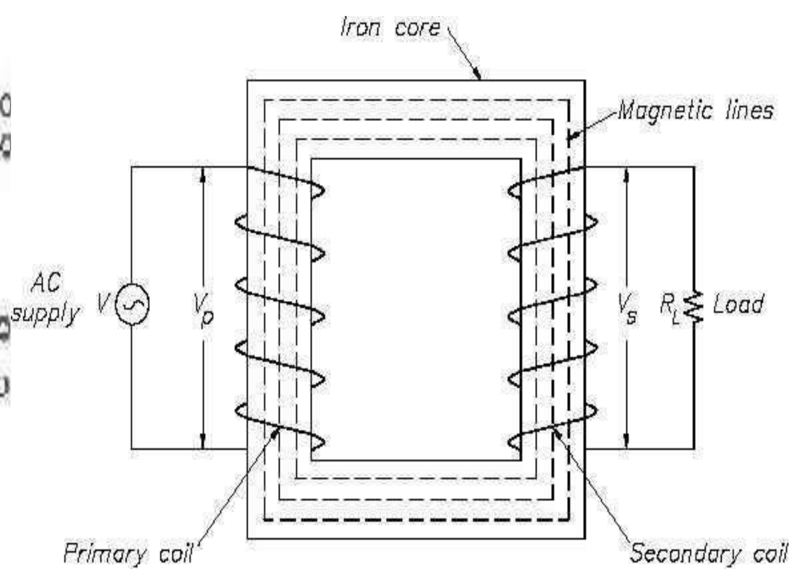


0 Rectangular core, two limbs.

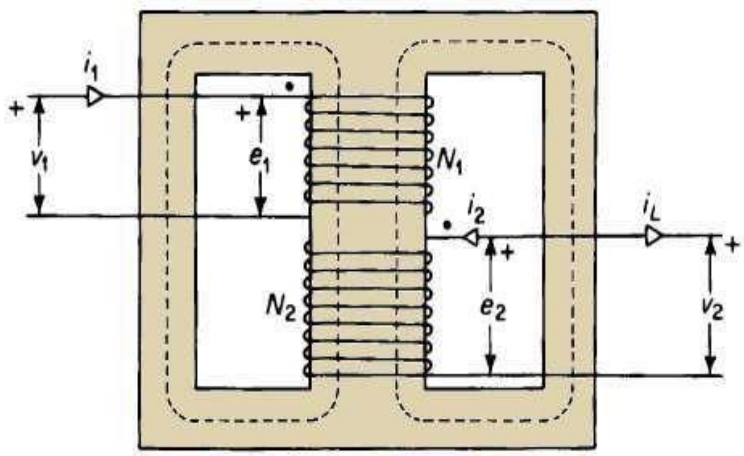
0 Winding encircles core and Low voltage coil is placed near the limb and insulation by paper and High voltage on it.

0 Windings are distributive type and natural cooling is effective and top laminations can be removed for maintenance work.

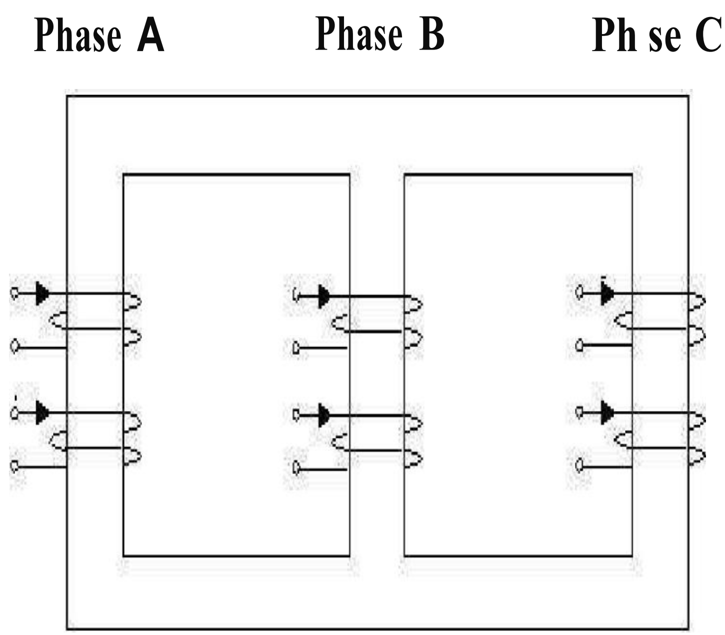




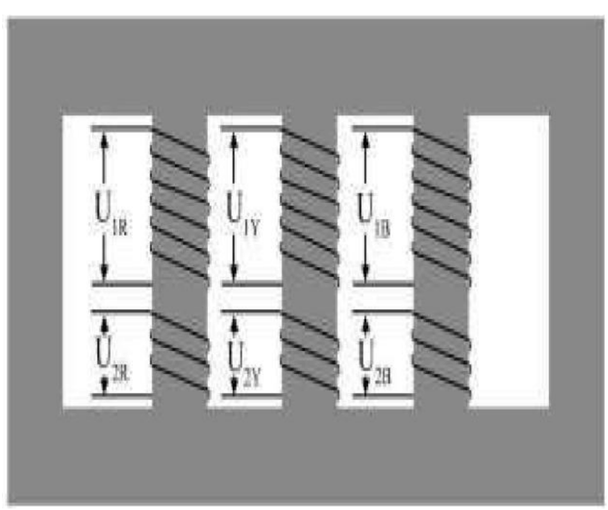
- Core Encircles most of windings
- Natural cooling is not possible
- Maintenance work is difficult
- For HV Transformers
- 1- cp requires three limbs
- Double magnetic circuit



- Core consists of three limbs, top and bottom yokes.
- Each limb consists of primary and secondary winding(LV and HV winding)
- Three phase transformer can also designed by arranging three single phase transformer in series.



- Shell type(five limb)is used for large transfonner because they can be made with a reduced height.
- The cost of three phase shell type transformer is more.
- For cooling of transformer fans are fixed at the radiators.



Core type

- Winding encircles core
- Cylindrical coils
- Natural cooling is effective
- Maintenance work is easy
- Single magnetic circuit
- Low Voltage and distribution type
- Two limbs for 1-phase and three for 3-phase

Shell type

- Core encircles windings
- Disc type

- Natural cooling is not effective
- Maintenance work is difficult
- Double magnetic circuit
- High Voltage transformer
- Three limbs for I-phase and 6-limbs for three phase

Types of Transformer

- Power Transformer
- Distribution Transformer
- Constant Voltage Transformer
- Constant Current Transformer
- Variable Frequency Transformer
- Auto Transformer

Power transformer of rating 500 mVA llkv/230v

- Transformer having rating more than 200kva is power transformers
- Usually this transformers are placed near the generating and substations to either step up or step down voltage levels
- The transformers which are used to transform the transmission voltage to the voltage level of primary feeders are called substation transformers

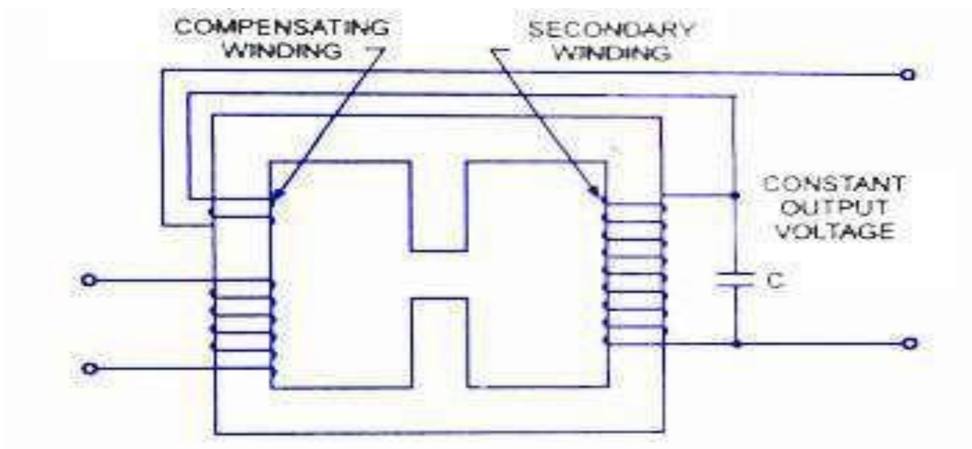
Fig: Power Transformer

Pad mounted & pole mounted distribution transformer

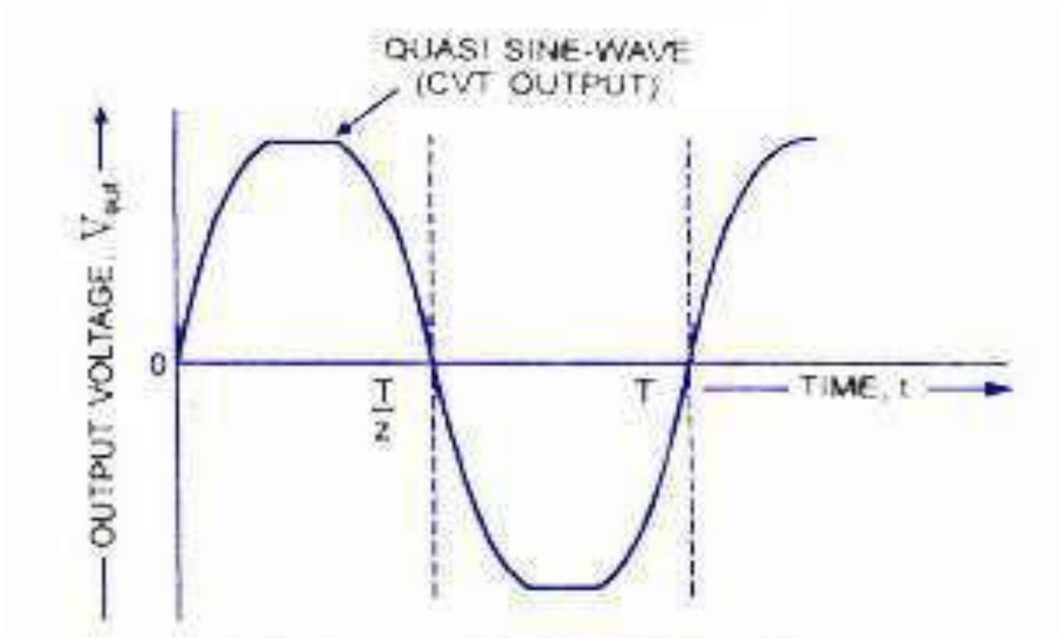
- It changes feeder voltage to the utilization voltage for customer requirements.
- This transformers operate throughout the day therefore iron loss will be throughout the day and copper loss occur only when it is loaded.
- These are low load high efficiency machines.
- It is designed in such way to maintain the small leakage reactance to get good voltage regulation as it want to operate throughout the day.
- Depending on the installation it is of pole mounted or pad mounted as shown in the diagram.

Constant voltage transformer and its output

- It uses the leakage inductance of its secondary windings in combination with external capacitors to create one or more resonant circuits.
- It consists of linear inductor which is unsaturated and this will be primary.
- The non linear inductor(saturated) forms the secondary of the transformer.
- The capacitor connected in parallel saturates by drawing the secondary current due to saturation a constant output voltage is produced.
- Since the output is a quasi sine wave because of the constant in output voltage and this is improved by the compensating winding.



(a) Contructional Details



(b) Output Voltage Waveform

Constant Current transformer

- It consists of Primary and secondary winding but one is movable and mounted on the same core
- A counter weight is used to balance the moving winding.
- The principle is production of two oppositely directed magnetic field
- If load impedance decreases load current increases due to this large opposition between two magnetic fields produced by primary and secondary
- Due to repulsion movable winding moves up and further gets separated from stationary and large leakage flux reduces and in tum mutual flux reduces thus secondary voltage reduces

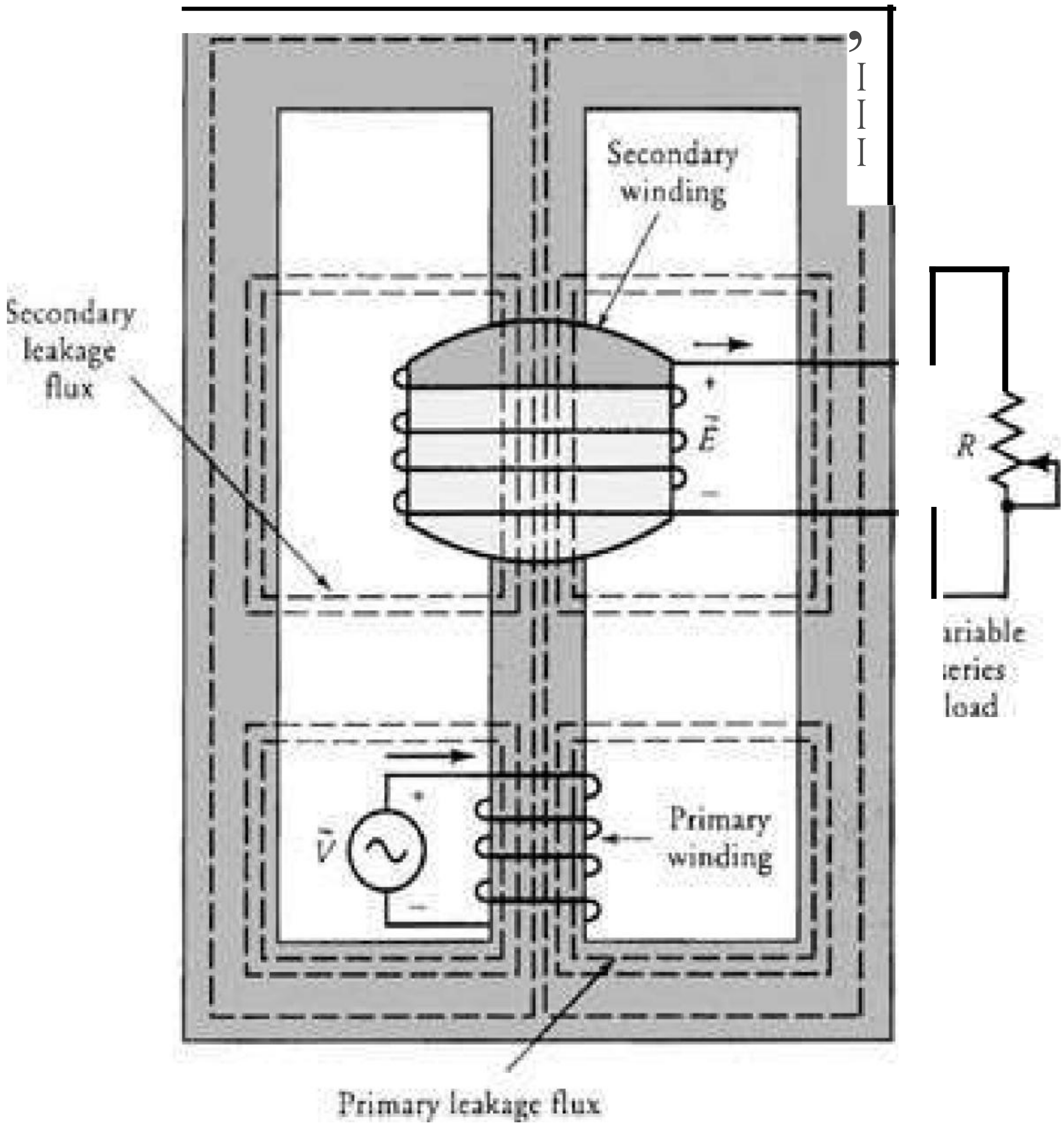


Fig: **Constant Current Transformer**

Variable frequency transformer

- The variable frequency transformer (VFT) is essentially a continuously variable phase shifting transformer that can operate at an adjustable phase angle
- A **variable frequency transformer** is used to transmit electricity between two asynchronous alternating current domains.
- A variable frequency transformer is a doubly-fed electric machine resembling a vertical shaft hydroelectric generator with a three-phase wound rotor, connected by slip rings to one external power circuit. A direct-current torque motor is mounted on the same shaft
- The phase shift between input and output voltage should also be small over the range of frequencies.
- The applications of VFT are Electronic circuits, Communication, Control and measurement which uses wide band of frequencies.

Losses in Transformer:

Losses of transformer are divided mainly into two types:

1. Iron Loss
2. Copper Losses

Iron Loss:

This is the power loss that occurs in the iron part. This loss is due to the alternating frequency of the emf. Iron loss in further classified into two other losses.

a) Eddy current loss **b) Hysterisis loss**

a) EDDY CURRENT LOSS: This power loss is due to the alternating flux linking the core, which will induced an emf in the core called the eddy emf, due to which a current called the eddy current is being circulated in the core. As there is some resistance in the core with this eddy current circulation converts into heat called the eddy current power loss. Eddy current loss is proportional to the square of the supply frequency.

b) HYSTERISIS LOSS: This is the loss in the iron core, due to the magnetic reversal of the flux in the core, which results in the form of heat in the core. This loss is directly proportional to the supply frequency.

Eddy current loss can be minimized by using the core made of thin sheets of silicon steel material, and each lamination is coated with varnish insulation to suppress the path of the eddy currents.

Hysterisis loss can be minimized by usmg the core material having high permeability.

Copper Loss:

This is the power loss that occurs in the primary and secondary coils when the transformer is on load. This power is wasted in the form of heat due to the resistance of the coils. This loss is proportional to the sequence of the load hence it is called the Variable loss where as the Iron loss is called as the Constant loss as the supply voltageand frequency are constants

Efficiency:

It is the ratio of the output power to the input power of a transformer

Input = Output + Total losses

= Output + Iron loss + Copper loss

Efficiency =

$$77 = \frac{outputpower}{outputpower + Ironloss + copperloss}$$

$$\frac{V_2 I_2 \cos ¢}{V_2 I_2 \cos\phi + W_{eron} + W_{copper}}$$

Where, V_2 is the secondary (output) voltage, lz is the secondary (output) current and cos<I> is the power factor of the load.

The transformers are normally specified with their ratings as KVA,

Therefore,

$$Efficiency; \eta = \frac{(KVA) \times 10^3 \times cos\varphi}{(KVA) \times 10^3 \times cos\varphi \times W_{iron} + W_{copper}}$$

Since the copper loss varies as the square of the load the efficiency of the transformer at any desired load *n* is given by

$$Efficiency; \eta = \frac{n \times (KVA) \times 10^3 \times cos\varphi}{n \times (KVA) \times 10^3 \times cos\varphi \times W_{iron} + n^2 \times W_{copper}}$$

where W_{copper} is the copper loss at full load

$W_{copper} = I^2R$ watts

CONDITION FOR MAXIMUM EFFICIENCY:

In general for the efficiency to be maximum for any device the losses must be minimum. Between the iron and copper losses the iron loss is the fixed loss and the copper loss is the variable loss. When these two losses are equal and also minimum the efficiency will be maximum.

Therefore the condition for maximum efficiency in a transformer is

Copper 1oss= Iro::n.10&1&1 (whichever is minimum)

VOLTAGE REGULATION:

The voltage regulation of a transformer is defined as the change in the secondary terminal voltage between no load and full load at a specified power factor expressed as a percentage of the full load terminal voltage.

$$\%Voltage\ Regulation = \frac{(no\ load\ Sec.Voltage) - (full\ load\ Sec.Voltage)}{full\ load\ Sec.Voltage} \times 100$$

Voltage regulation is a measure of the change in the terminal voltage of a transformer between No load and Full load. A good transformer has least value of the regulation of the order of **±5%**

If a load is connected to the secondary, an electric current will flow in the secondary winding and electrical energy will be transferred from the primary circuit through the transformer to the load. In an ideal transformer, the induced voltage in the secondary winding (Vs) is in proportion to the primary voltage (Vp), and is given by the ratio of the number of turns in the secondary (Ns) to the number of turns in the primary (Np) as follows:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}$$

Earlier it is seen that a voltage is induced in a coil when the flux linkage associated with the same changed. If one can generate a time varying magnetic field any coil placed in the field of influence linking the same experiences an induced emf. A time varying field can be created by passing an alternating current through an electric coil. This is called mutual induction. The medium can even be air. Such an arrangement is called air cored transformer.

Indeed such arrangements are used in very high frequency transformers. Even though the principle of transformer action is not changed, the medium has considerable influence on the working of such devices. These effects can be summarized as the followings.

1. The magnetizing current required to establish the field is very large, as the reluctance of the medium is very high.

2. There is linear relationship between the mmf created and the flux produced.

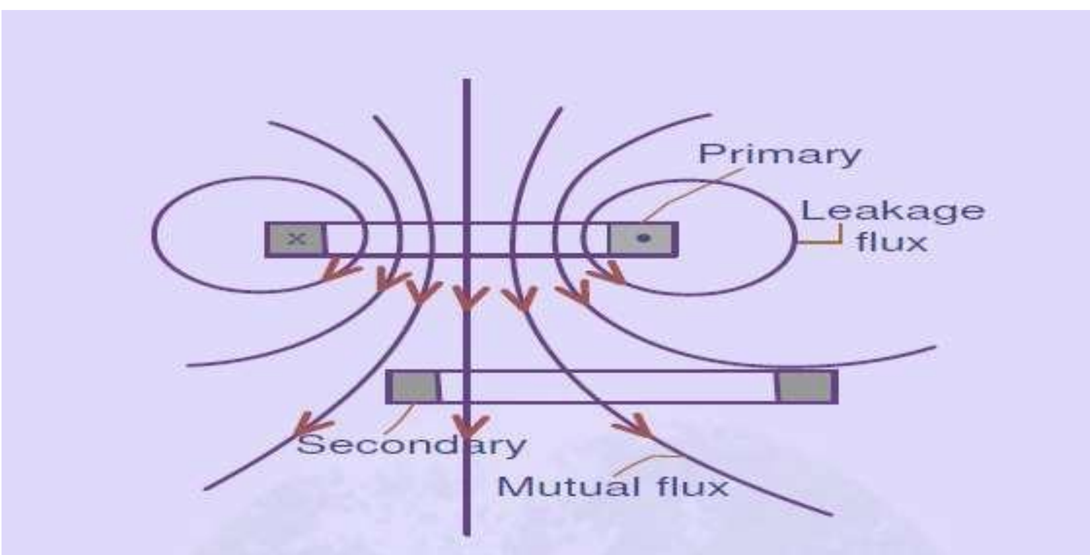
3. The medium is non-lossy and hence no power is wasted in the medium.

4. Substantial amount of leakage flux exists.

5. It is very hard to direct the flux lines as we desire, as the whole medium is homogeneous.

If the secondary is not loaded the energy stored in the magnetic field finds its way back to the source as the flux collapses. If the secondary winding is connected to a load then part of the power from the source is delivered to the load through the magnetic field as a link.

The medium does not absorb and lose any energy. Power is required to create the field and not to maintain the same. As the winding losses can be made very small by proper choice of material, the ideal efficiency of a transformer approaches 100%. The large magnetizing current requirement is a major deterrent.



(a)

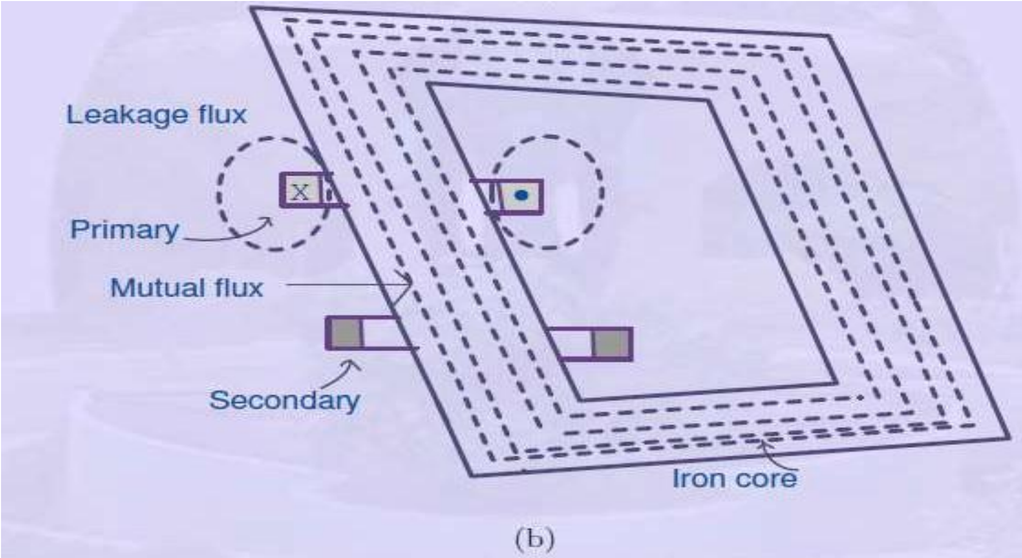


Figure 10: Mutual Induction a) air core b) iron core

1. Due to the large value for the permeance (μr of the order of 1000 as compared to air) the magnetizing current requirement decreases dramatically. This can also be visualized as a dramatic increase in the flux produced for a given value of magnetizing current.

2. The magnetic medium is linear for low values of induction and exhibits saturation type of non-linearity at higher flux densities.

3. The iron also has hysteresis type of non-linearity due to which certain amount of power is lost in the iron (in the form of hysteresis loss), as the B H characteristic is traversed.

4. Most of the flux lines are confined to iron path and hence the mutual flux is increased very much and leakage flux is greatly reduced.

5. The flux can be easily 'directed' as it takes the path through steel which gives great freedom for the designer in physical arrangement of the excitation and output windings.

6. As the medium is made of a conducting material eddy currents are induced in the same and produce losses. These are called 'eddy current losses'. To minimize the eddy current losses the steel core is required to be in the form of a stack of insulated laminations.

From the above it is seen that the introduction of magnetic core to carry the flux introduced two more losses. Fortunately the losses due to hysteresis and eddy current for the available grades of steel are very small at power frequencies. Also the copper losses in the winding due to magnetization current are reduced to an almost insignificant fraction of the full load losses. Hence steel core is used in power transformers.

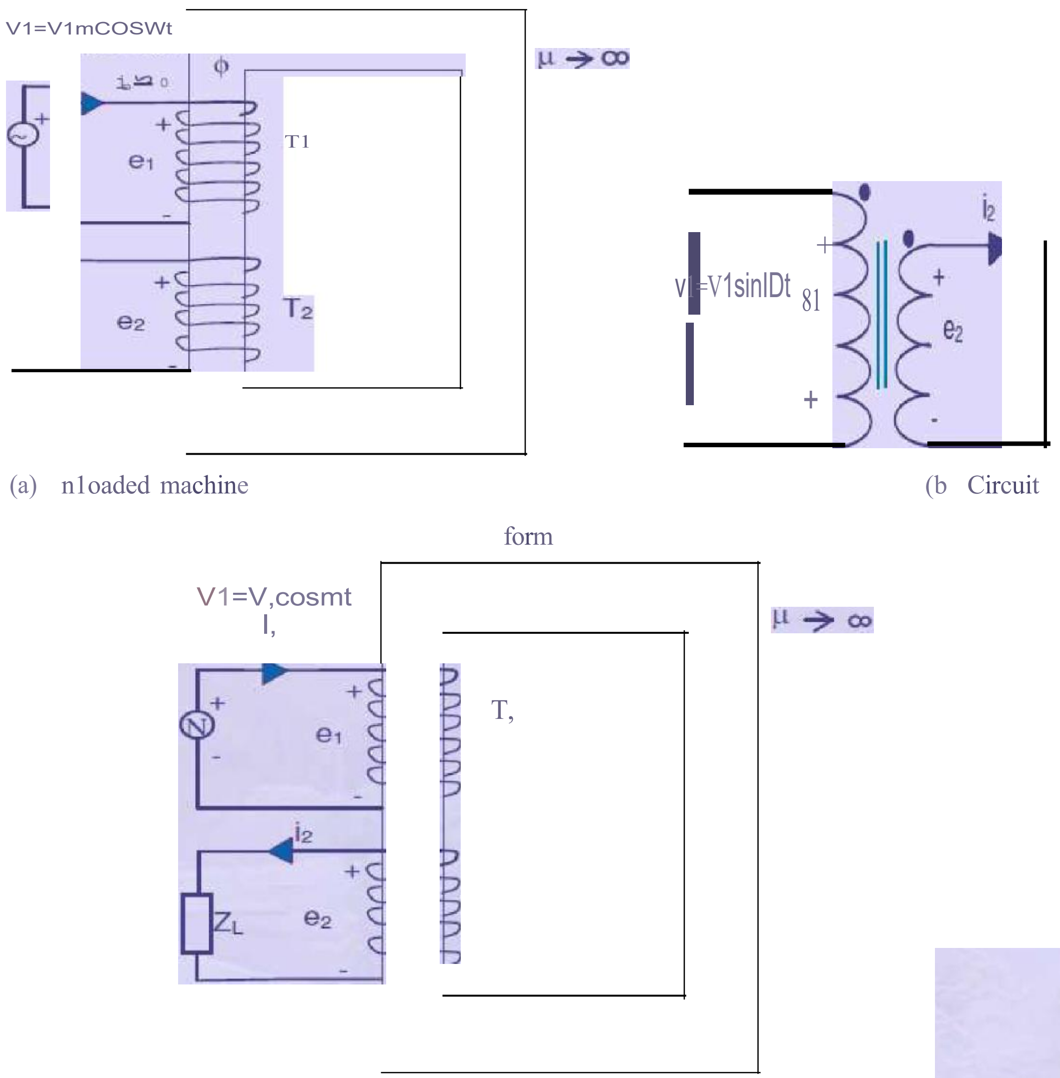
In order to have better understanding of the behavior of the transformer, initially certain idealizations are made and the resulting 'ideal' transformer is studied. These idealizations are as follows:

1. Magnetic circuit is linear and has infinite permeability. The consequence is that a vanishingly small current is enough to establish the given flux. Hysteresis loss is negligible. As all the flux generated confines itself to the iron, there is no leakage flux.

2. Windings do not have resistance. This means that there are no copper losses, nor there is any ohmic drop in the electric circuit.

In fact the practical transformers are very close to this model and hence no major departure is made in making these assumptions. Fig 11 shows a two winding ideal transformer. The primary winding has T, turns and is connected to a voltage source of V, volts. The secondary has T_2 turns. Secondary can be connected to load impedance for loading the transformer. The primary and secondary are shown on the same limb and separately for clarity.

As a current Io amps is passed through the primary winding of T1 turns it sets up an MMF ofioTl ampere which is in tum sets up a flux_ through the core. Since the reluctance of the iron path given by R = 1/µA is zero as µ -1, a vanishingly small value of current Io is enough to setup a flux which is finite. As Io establishes the field inside the transformer



(a) n1oaded machine (b Circuit form

( )Load d machine

Figure 11: Two winding Ideal Transformer unloaded and loaded

it is called the magnetizing current of the transformer.

$$lux \quad \phi = \frac{mmf}{Reluctanc} = \frac{i_0 T_1}{l / \mu \cdot l} = \frac{i_0 T_1 \mu}{l}$$

This current is the result of a sinusoidal voltage V applied to the primary. As the current through the loop is zero (or vanishingly small), at every instant of time, the sum of the voltages must be zero inside the same. Writing this in terms of instantaneous values we have, v1 - e1 = 0 where v1 is the instantaneous value of the applied voltage and el is the induced emf due to Faradays principle. The negative sign is due to the application of the Lenz's law and shows that it is in the form of a voltage drop. Kirchoffs law application to the loop will result in the same thing.

This equation results in v1 = e1 or the induced emf must be same in magnitude to the applied voltage at every instant of time. Let v1 = V!peak cosmt where V1peak is the peak value and m= *2nf* t. f is the frequency of the supply. As v1 = e1; e1 = d 1/dt but e1 = E1peak cosmt) E1 = V1 . It can be easily seen that the variation of flux linkages can be obtained as \j/1 = \jflpeak sinmt. Here \j/Jpeak is the peak value of the flux linkages of the primary.

Thus the RMS primary induced EMF is

$$
\begin{aligned}
e_1 &= \frac{d\psi_1}{dt} = \frac{d(\psi_{1peak} \sin \omega t)}{dt} \\
&= \psi_{1peak}.\omega.\cos \omega t \quad \text{or the rms value} \\
E_1 &= \frac{\psi_{1peak}.\omega}{\sqrt{2}} = \frac{2\pi f T_1 \phi_m}{\sqrt{2}} = 4.44 f \phi_m T_1 \quad \text{volts}
\end{aligned}
$$

Here \jflpeak is the peak value of the flux linkages of the primary. The same mutual flux links the secondary winding. However the magnitude of the flux linkages will be \jf2peak =T2<pm. The induced emf in the secondary can be similarly obtained as

$$
\begin{aligned}
e_2 &= \frac{d\psi_2}{dt} = \frac{d(\psi_{2peak} \sin \omega t)}{dt} \\
&= \psi_{2peak}.\omega.\cos \omega t \quad \text{or the rms value} \\
E_2 &= \frac{2\pi f T_2 \phi_m}{\sqrt{2}} = 4.44 f \phi_m T_2 \quad \text{volt}
\end{aligned}
$$

Which yields the voltage ratio as El/E2=Tl/T2

Transformer at loaded condition.

So far, an unloaded ideal transformer is considered. If now a load impedance ZL is connected across the terminals of the secondary winding a load current flows as marked in Fig. 1l(c).This load current produces a demagnetizing mmf and the flux tends to collapse. However this is detected by the primary immediately as both E2 and E1 tend to collapse.

The current drawn from supply increases up to a point the flux in the core is restored back to its original value. The demagnetizing mmf produced by the secondary is neutralized by additional magnetizing mmf produces by the primary leaving the mmf and flux in the core as in the case of no-load. Thus the transformer operates under constant induced emf mode. Thus

$$\begin{aligned} i_1T_1 - i_2T_2 &= i_0T_1 \quad \text{but} \quad i_0 \rightarrow 0 \\ i_2T_2 &= i_1T_1 \quad \text{and the rms value} \quad I_2T_2 = I_1T_1. \end{aligned}$$

If the reference directions for the two currents are chosen as in the Fig. 12, then the above equation can be written in phasor form as,

$$\begin{aligned} \bar{I}_1T_1 &= \bar{I}_2T_2 \quad \text{or} \quad \bar{I}_1 = \frac{T_2}{T_1}.\bar{I}_2 \\ \text{Also} \quad \frac{E_1}{E_2} &= \frac{T_1}{T_2} = \frac{I_2}{I_1} \quad E_1I_1 = E_2I_2 \end{aligned}$$

Thus voltage and current transformation ratio are inverse of one another. If an impedance of ZL is connected across the secondary,

$$\bar{Z}_i = \frac{\bar{E}_1}{\bar{I}_1} = (\frac{T_1}{T_2})^2.\frac{\bar{E}_2}{\bar{I}_2} = (\frac{T_1}{T_2})^2.\bar{Z}_L$$

An impedance of ZL when viewed 'through' a transformer of turns ratio (Tl/T2) is seen as (Tl/T2)2.ZL. Transformer thus acts as an impedance converter. The transformer can be interposed **in** between a source and a load to 'match' the impedance.

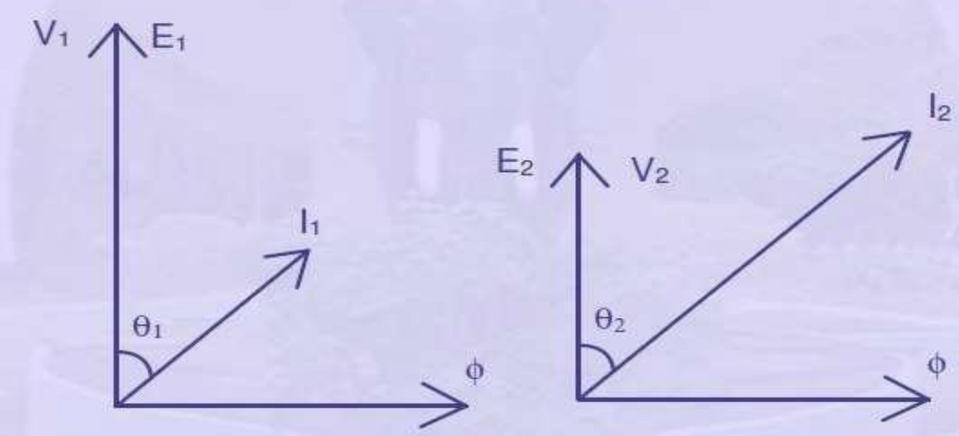


Figure 13: Phasor diagram of Operation of an Ideal Transformer

Finally, the phasor diagram for the operation of the ideal transformer is shown in

Fig. 13 in which 0 **1** and 0 2 are power factor angles on the primary and secondary sides. As the transfonner itself does not absorb any active or reactive power **it** is easy to see that 01 = 02.

Thus, from the study of the ideal transformer it is seen that the transformer provides electrical isolation between two coupled electric circuits while maintaining power invariance at its two ends. However, grounding of loads and one terminal of the transformer on the secondary/primary side are followed with the provision of leakage current detection devices to safe guard the persons working with the devices. Even though the isolation aspect is a desirable one its utility cannot be over emphasized. It can be used to step up or step down the voltage/current at constant volt-ampere. Also, the transformer can be used for impedance matching. In the case of an ideal transformer the efficiency is 100% as there are no losses inside the device.

Practical Transformer

An ideal transformer is useful in understanding the working of a transformer. But it cannot be used for the computation of the performance of a practical transformer due to the non-ideal nature of the practical transformer. In a working transformer the performance aspects like magnetizing current, losses, voltage regulation, efficiency etc are important. Hence the effects of the non-idealization like finite permeability, saturation, hysteresis and winding resistances have to be added to an ideal transformer to make it a practical transformer.

Conversely, if these effects are removed from a working transformer what is left behind is an ideal transformer.

Finite penneability of the magnetic circuit necessitates a finite value of the current to be drawn from the mains to produce the mmf required to establish the necessary flux.

The current and mmf required is proportional to the flux density B that is required to be established in the core.

$$B = \mu H; \quad B = \frac{\phi}{A}$$

where A is the area of cross section of the iron core m2. **H** is the magnetizing force which is given by,

$$H = i.\frac{T_1}{l}$$

where l is the length of the magnetic path, m. or

$$\phi = B.A = \frac{A\mu(iT_1)}{l} \quad = \quad permeance * mmf(here \quad that \quad of \quad primary)$$

The magnetizing force and the current vary linearly with the applied voltage as long as the magnetic circuit is not saturated. Once saturation sets in, the current has to vary in a nonlinear manner to establish the flux of sinusoidal shape. This non-linear current can be resolved into fundamental and harmonic currents. This is discussed to some extent under harmonics. At present the effect of this non-linear behavior is neglected as a secondary effect. Hence the current drawn from the mains is assumed to be purely sinusoidal and directly proportional to the flux density of operation. This current can be represented by a current drawn by an inductive reactance in the circuit as the net energy associated with the same over a cycle is zero. The energy absorbed when the current increases are returned to the electric circuit when the current collapses to zero. This current is called the magnetizing current of the transformer. The magnetizing current Im is given by Im= El/Xm where Xm is called the magnetizing reactance. The magnetic circuit being lossy absorbs and dissipates the power depending upon the flux density of operation. These losses arise out of hysteresis, eddy current inside the magnetic core. These are given by the following express10ns:

$$P_h \propto B^{1.6} f$$

$$P_e \propto B^2 f^2 t^2$$

Ph -Hysteresis loss, Watts

B- Flux density of operation Tesla.

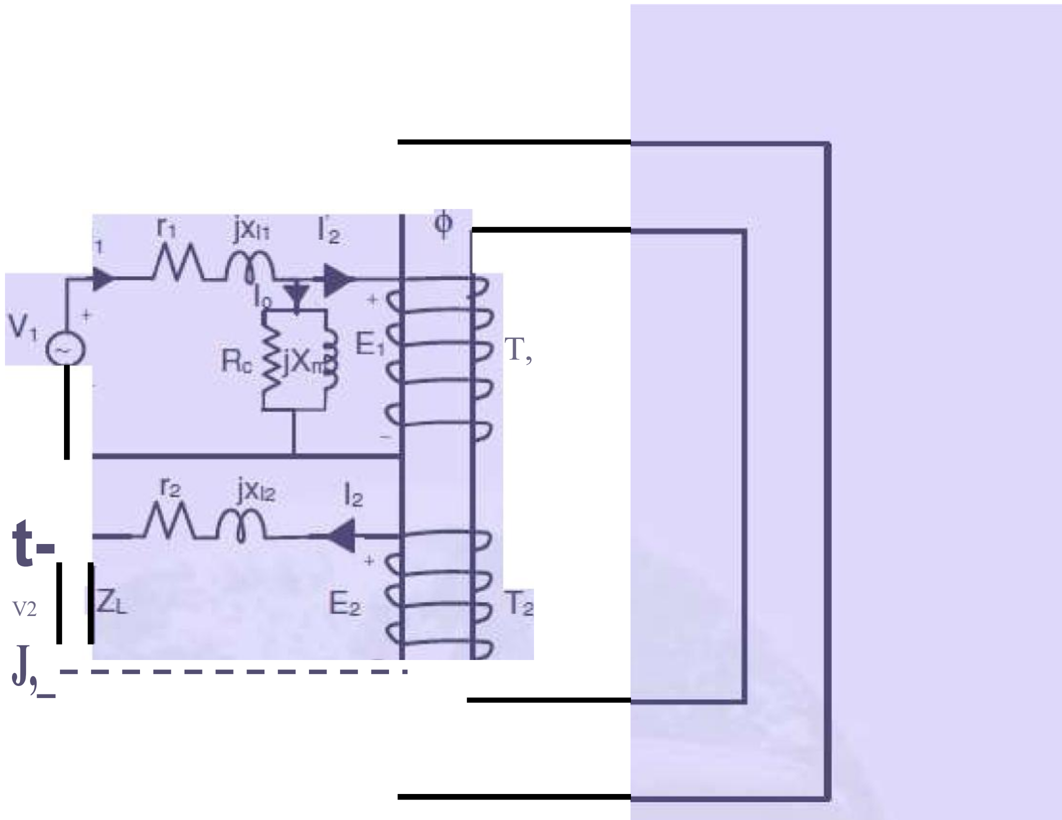
f - Frequency of operation, Hz

t - Thickness of the laminations of the core, m.

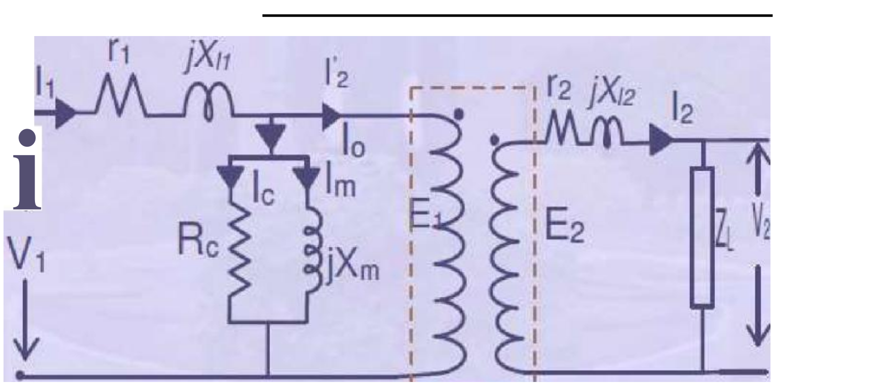
For a constant voltage, constant frequency operation B is constant and so are these losses. An active power consumption by the no-load current can be represented in the input circuit as a resistance Re connected in parallel to the magnetizing reactance Xm. Thus the no-load current IO may be made up of Ie (loss component) and Im (magnetizing component as) Io = le - jlmI^2cRc- gives the total core losses (i.e. hysteresis + eddy current loss) I^2mXm- Reactive volt amperes consumed for establishing the mutual flux.

Finite µ of the magnetic core makes a few lines of flux take to a path through the air. Thus these flux lines do not link the secondary winding. It is called as leakage flux. As the path of the leakage flux is mainly through the air the flux produced varies linearly with the primary current I,. Even a large value of the current produces a small value of flux. This flux produces a voltage drop opposing its cause, which is the current I,. Thus this effect of the finite permeability of the magnetic core can be represented as a series inductive element jxl,. This is termed as the reactance due to the primary leakage flux. As this leakage flux varies linearly with Ii, the flux linkages per ampere and the primary leakage inductance are constant (This is normally represented by lli Henry). The primary leakage reactance therefore becomes xl, = 2nfll, ohm.

A similar effect takes place on the secondary side when the transformer is loaded. The secondary leakage reactance jxh arising out of the secondary leakage inductance lb is given by xh = 2nflh Finally, the primary and secondary windings are wound with copper (sometimes aluminum in small transformers) conductors; thus the windings have a finite resistance (though small). This is represented as a series circuit element, as the power lost and the drop produced in the primary and secondary are proportional to the respective currents. These are represented by r, and r2 respectively on primary and secondary side. A practical transformers ans these imperfections (taken out and represented explicitly in the electric circuits) is an ideal transformer of turns ratio T, : T2 (voltage ratio E, : Ei). This is seen in Fig. 14. I'2 in the circuit represents the primary current component that is required to flow from the mains in the primary T_1 turns to neutralize the demagnetizing secondary current Ii due to the load in the secondary turns. The total primary current



(.a)Physical-arrang \illen



(b) Eqwvalent circuit

Figme 14: A Practical Tr&nsformer

$$\text{vectorially is} \quad \bar{I}_1 = \bar{I}_2' + \bar{I}_0$$

$$\text{Here} \quad I_2'T_1 = I_2T_2 \quad \text{or} \quad I_2' = I_2\frac{T_2}{T_1}$$

$$\text{Thus} \quad \bar{I}_1 = \bar{I}_2\frac{T_2}{T_1} + \bar{I}_0$$

By solving this circuit for any load impedance ZL one can find out the performance of the loaded transformer.

The circuit shown **in** Fig. 14(b). However, it is not very convenient for use due to the presence of the ideal transformer of turns ratio **T**1 : T2. If the turns ratio could be made unity by some transformation the circuit becomes very simple to use. This is done here by replacing the secondary by a 'hypothetical' secondary having T1 turns which is 'equivalent' to the physical secondary. The equivalence implies that the ampere turns, active and reactive power associated with both the circuits must be the same. Then there is no change as far as their effect on the primary is considered.

Thus

$$V_2' = aV_2, \quad I_2' = \frac{I_2}{a}, \quad r_2' = a^2 r_2, \quad x_{l2}' = a^2 x_{l2} \quad Z_L' = a^2 Z_L.$$

where a -turns ratio Ti/T2

As the ideal transformer in this case has a turns ratio of unity the potentials on either side are the same and hence they may be conductively connected dispensing away with the ideal transformer. This particular equivalent circuit is as seen from the primary side. It is also possible to refer all the primary parameters to secondary by making the hypothetical equivalent primary winding on the input side having the number of turns to be T2. Such an equivalent circuit having all the parameters referred to the secondary side is shown in fig.

The equivalent circuit can be derived, with equal ease, analytically using the Kirchoffs equations applied to the primary and secondary. Referring to fig. 14(a), we have (by neglecting the shunt branch)

$$
\begin{aligned}
V_1 &= E_1 + I_1(r_1 + jx_{l1}) \\
E_2 &= V_2 + I_2(r_2 + jx_{l2}) \\
T_1 I_0 &= T_1 I_1 + T_2 I_2 \quad \text{or} \quad I_1 = -\frac{I_2}{a} + I_0 \\
&= -\frac{I_2}{a} + I_c + I_m \\
a &= \frac{T_1}{T_2}.
\end{aligned}
$$

Multiply both sides of Eqn.34 by 'a' [This makes the turns ratio unity and retains the power invariance].

$$
aE_2 = aV_2 + aI_2(r_2 + jx_{l2}) \qquad \text{but} \quad aE_2 = E_1
$$

Substituting in Eqn we have

$$
\begin{aligned}
V_1 &= aV_2 + aI_2(r_2 + jx_{l2}) + I_1(r_1 + jx_{l1}) \\
&= V_2' + I_1(a^2 r_2 + ja^2 x_{l2}) + I_1(r_1 + jx_{l1}) \\
&= V_2' + I_1(\overline{r_1 + r_2'} + j\overline{x_{l1} + x_{l2}'})
\end{aligned}
$$

A similar procedure can be used to refer all parameters to secondary side. (Shown in fig)

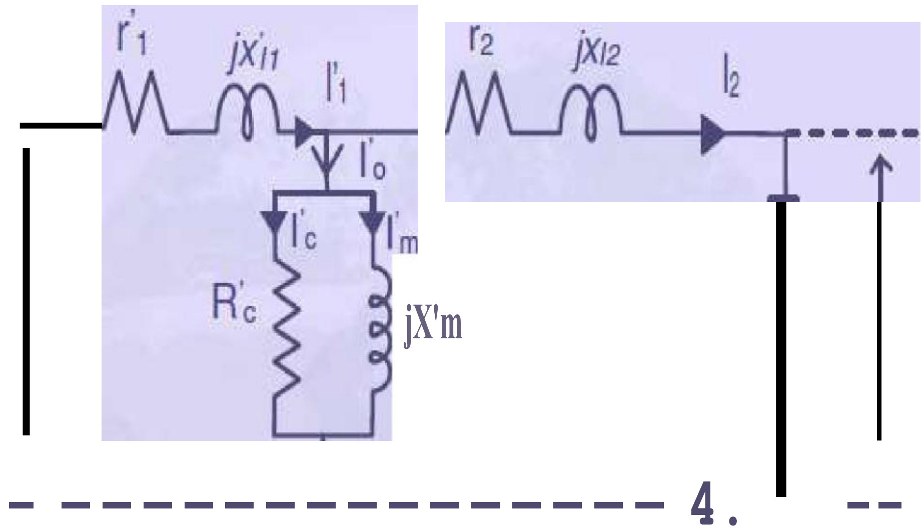


Figure rn: Equivalent, Circuit Referred o 11 ndary ide

Phasor Diagrams

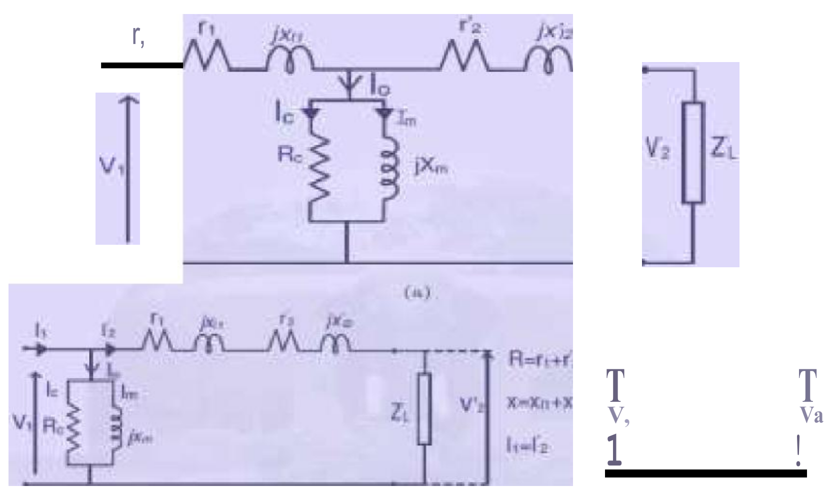
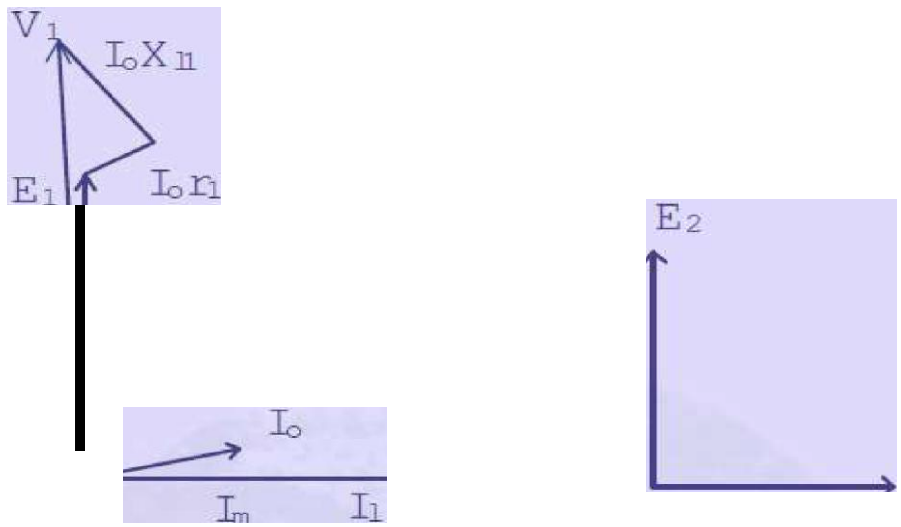


Figure 16: Exact,approximat ••I ITJJ p1itJ.c,J JUi-.W t • Lr• •Ul

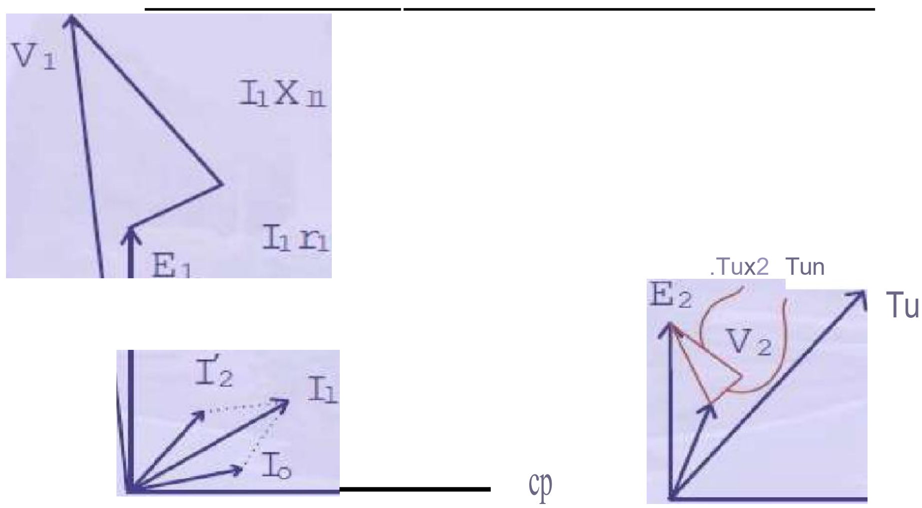
The resulting equivalent circuit as shown in Fig. 16 is known as the exact equivalent circuit. This circuit can be used for the analysis of the behavior of the transformers. As the no-load current is less than 1% of the load current a simplified circuit known as 'approximate' equivalent circuit (see Fig. 16(b)) is usually used, which may be further simplified to the one shown in Fig. 16(c).

On similar lines to the ideal transformer the phasor diagram of operation can be drawn for a practical transformer also. The positions of the current and induced emf phasor are not known uniquely if we start from the phasor V1. Hence it is assumed that the phasor is known. The E, and fo phasor are then uniquely known. Now, the magnetizing and loss components of the currents can be easily represented. Once Io is known, the drop that takes place in the primary resistance and series reactance can be obtained which when added to E1 gives uniquely the position of V1 which satisfies all other parameters. This is represented in Fig. 17(a) as phasor diagram on no-load.

Next we proceed to draw the phasor diagram corresponding to a loaded transformer. The position of the fo vector is known from the flux phasor. Magnitude of b and the load power factor angle 82 are assumed to be known. But the angle 82 is defined with respect to the terminal voltage V2 and not fo. By trial and error the position of b and V2 are determined. V2 should also satisfy the Kirchoff's equation for the secondary. Rest of the construction of the phasor diagram then becomes routine. The equivalent primary current I'2 is added vectorially to Io to yield I1. I1(ri+jxli)is added to E1 to yield V1. This is shown in fig. 17(b) as phasor diagram for a loaded transformer.



aJNo-Joad



(b On-load

Figure 17: Phasor Diagram of a. Prac ioal Tran:oformer

Open Circuit Test

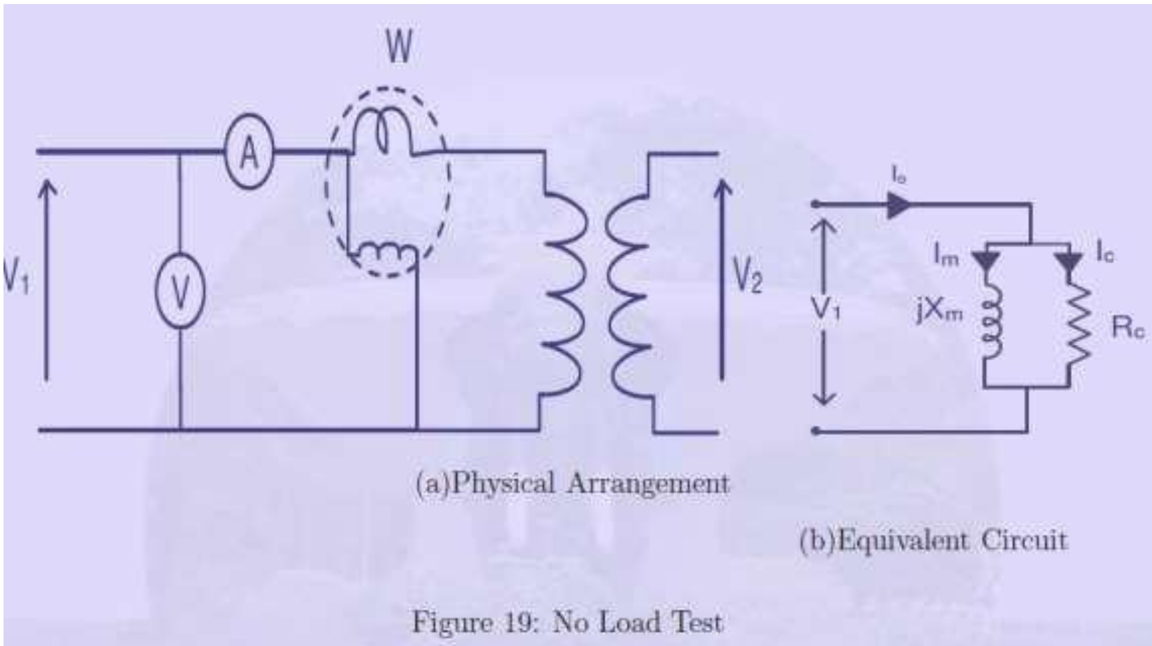


Figure 19: No Load Test

As the name suggests, the secondary is kept open circuited and nominal value of the input voltage is applied to the primary winding and the input current and power are measured. In Fig. 19(a) V,A,W are the voltmeter, ammeter and wattmeter respectively. Let these meters read V1, IO and WO respectively.Fig. 19(b) shows the equivalent circuit of the transformer under this test. The no load current at rated voltage is less than 1 percent of nominal current and hence the loss and drop that take place in primary impedance r1 +jxll due to the no load current IO is negligible. The active component Iw of the no load current IO represents the core losses and reactive current Im is the current needed for the magnetization.

Thus the watt meter reading

$$
\begin{aligned}
W_0 &= V_1 I_c = P_{core} \\
\therefore I_c &= \frac{W_0}{V_1} \\
\therefore I_m &= \sqrt{I_0^2 - I_c^2} \quad \text{or} \\
R_c &= \frac{V_1}{I_c} \quad \text{and} X_m = \frac{V_1}{I_m}
\end{aligned}
$$

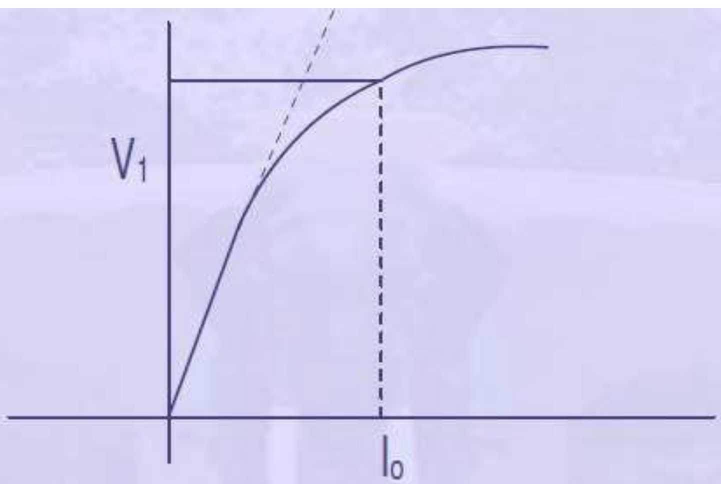


Figure 20: Open Circuit Characteristics

The parameters measured already are in terms of the pnmary. Sometimes the primary voltage required may be in kilo-Volts and it may not be feasible to apply nominal voltage to primary from the point of safety to personnel and equipment. If the secondary voltage is low, one can perform the test with LV side energized keeping the HV side open circuited. In this case the parameters that are obtained are in terms of LV . These have to be referred to HV side if we need the equivalent circuit referred to HV side.

Sometimes the nominal value of high voltage itself may not be known, or in doubt, especially in a rewound transformer. In such cases an open circuit characteristics is first obtained, which is a graph showing the applied voltage as a function of the no load current.

This is a non linear curve as shown in Fig. 20. This graph is obtained by noting the current drawn by transformer at different applied voltage, keeping the secondary open circuited. The usual operating point selected for operation lies at some standard voltage around the knee point of the characteristic. After this value is chosen as the nominal value the parameters are calculated as mentioned above.

Short Circuit Test

The purpose of this test 1s to determine the senes branch parameters of the equivalent circuit of Fig. 21(b). As the name suggests, in this test primary applied voltage, the current and power input are measured keeping the secondary terminals short circuited. Let these values be Vsc, Isc and Wsc respectively. The supply voltage required to circulate rated current through the transformer is usually very small and is of the order of a few percent of the nominal voltage. The excitation current which is only 1 percent or less even at rated voltage becomes negligibly small during this test and hence is neglected. The shunt branch is thus assumed to be absent. Also 11 = 12 as IO 0. Therefore Wsc is the sum of the copper losses in primary and secondary put together. The reactive power consumed is that absorbed by the leakage reactance of the two windings.

$$W_{sc} = I_{sc}^2(r_1 + r_2')$$

$$Z_{sc} = \frac{V_{sc}}{I_{sc}}$$

$$(x_{l1} + x_{l2}') = \sqrt{Z_{sc}^2 - (r_1 + r_2')^2}$$

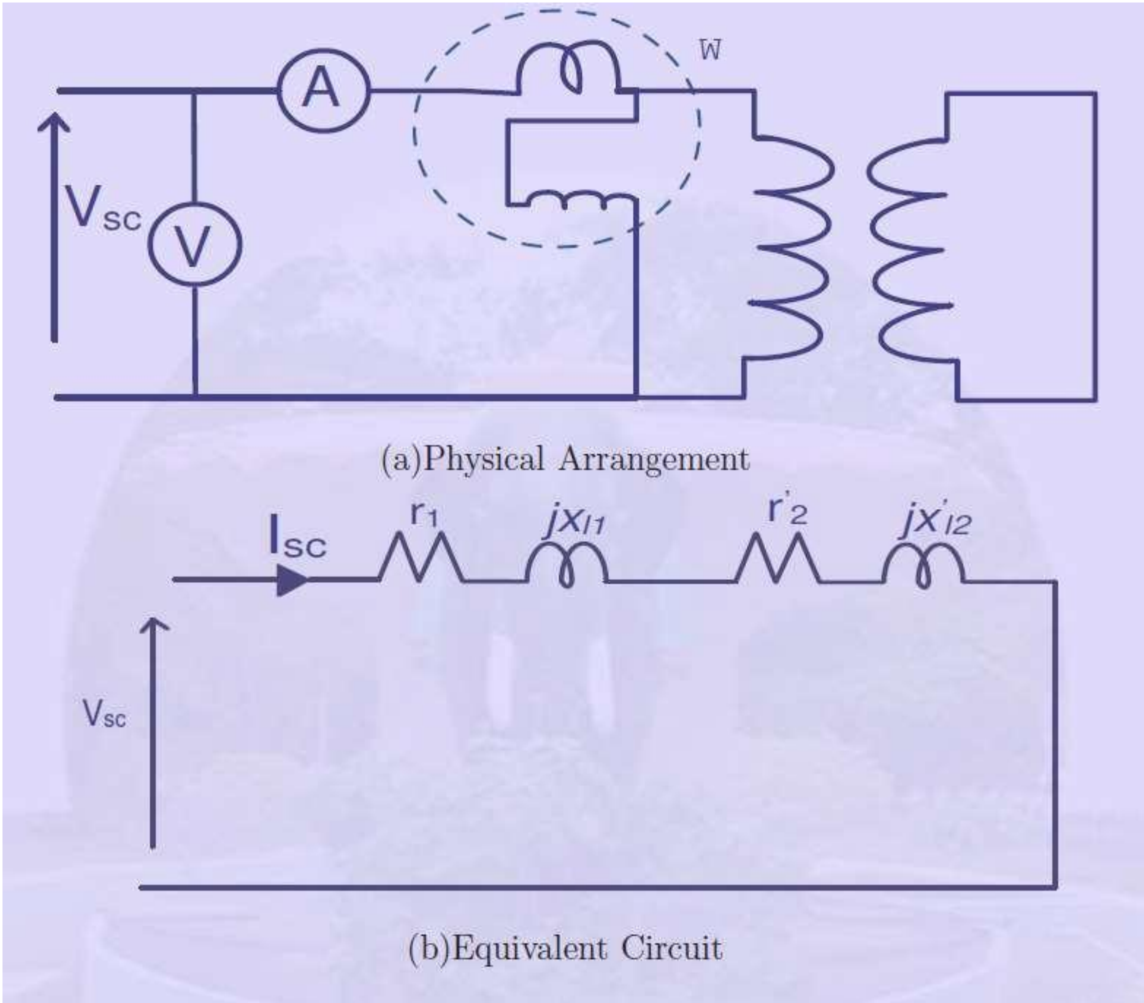


Figure 21: Short Circuit Test

If the approximate equivalent circuit is required then there is no need to separate r' 1 and r'2 or xll and x'l2. However if the exact equivalent circuit is needed then either r' 1 or'2 is determined from the resistance measurement and the other separated from the total.

As for the separation of xll and x'l2 is concerned, they are assumed to be equal. This is a fairly valid assumption for many types of transformer windings as the leakage flux paths are through air and are similar.

Load Test

Load Test helps to determine the total loss that takes place, when the transformer is loaded. Unlike the tests described previously, in the present case nominal voltage is applied across the primary and rated current is drown from the secondary. Load test is used mainly

1. to determine the rated load of the machine and the temperature rise

2. to determine the voltage regulation and efficiency of the transformer.

Rated load is determined by loading the transformer on a continuous basis and observing the steady state temperature rise. The losses that are generated inside the transformer on load appear as heat. This heats the transformer and the temperature of the transformer increases. The insulation of the transformer is the one to get affected by this rise in the temperature. Both paper and oil which are used for insulation in the transformer start getting degenerated and get decomposed. If the flash point of the oil is reached the transformer goes up in flames. Hence to have a reasonable life expectancy the loading of the transformer must be limited to that value which gives the maximum temperature rise tolerated by the insulation. This aspect of temperature rise cannot be guessed from the electrical equivalent circuit. Further, the losses like dielectric losses and stray load losses are not modeled in the equivalent circuit and the actual loss under load condition will be in error to that extent.

Many external means of removal of heat from the transformer in the form of different cooling methods give rise to different values for temperature rise of insulation. Hence these permit different levels of loading for the same transformer. Hence the only sure way of ascertaining the rating is by conducting a load test. It is rather easy to load a transformer of small ratings. As the rating increases it becomes difficult to find a load that can absorb the requisite power and a source to feed the necessary current. As the transformers come in varied transformation ratios, in many cases it becomes extremely difficult to get suitable load impedance.

Further, the temperature rise of the transformer is due to the losses that take place 'inside' the transformer. The efficiency of the transformer is above 99% even in modest sizes which means 1 percent of power handled by the transformer actually goes to heat up the machine. The remaining 99% of the power has to be dissipated in a load impedance external to the machine. This is very wasteful in terms of energy also. (If the load is of unity power factor) Thus the actual loading of the transformer is seldom resorted to. Equivalent loss methods of loading and 'Phantom' loading are commonly used in the case of transformers.

The load is applied and held constant till the temperature rise of transformer reaches a steady value. If the final steady temperature rise is lower than the maximum permissible value, then load can be increased else it is decreased. That load current which gives the maximum permissible temperature rise is declared as the nominal or rated load current and the volt amperes are computed using the same.

In the equivalent loss method a short circuit test is done on the transformer. The short circuit current is so chosen that the resulting loss taking place inside the transformer is equivalent to the sum of the iron losses, full load copper losses and assumed stray load losses. By this method even though one can pump in equivalent loss inside the transformer, the actual distribution of this loss vastly differs from that taking place in reality. Therefore this test comes close to a load test but does not replace one.

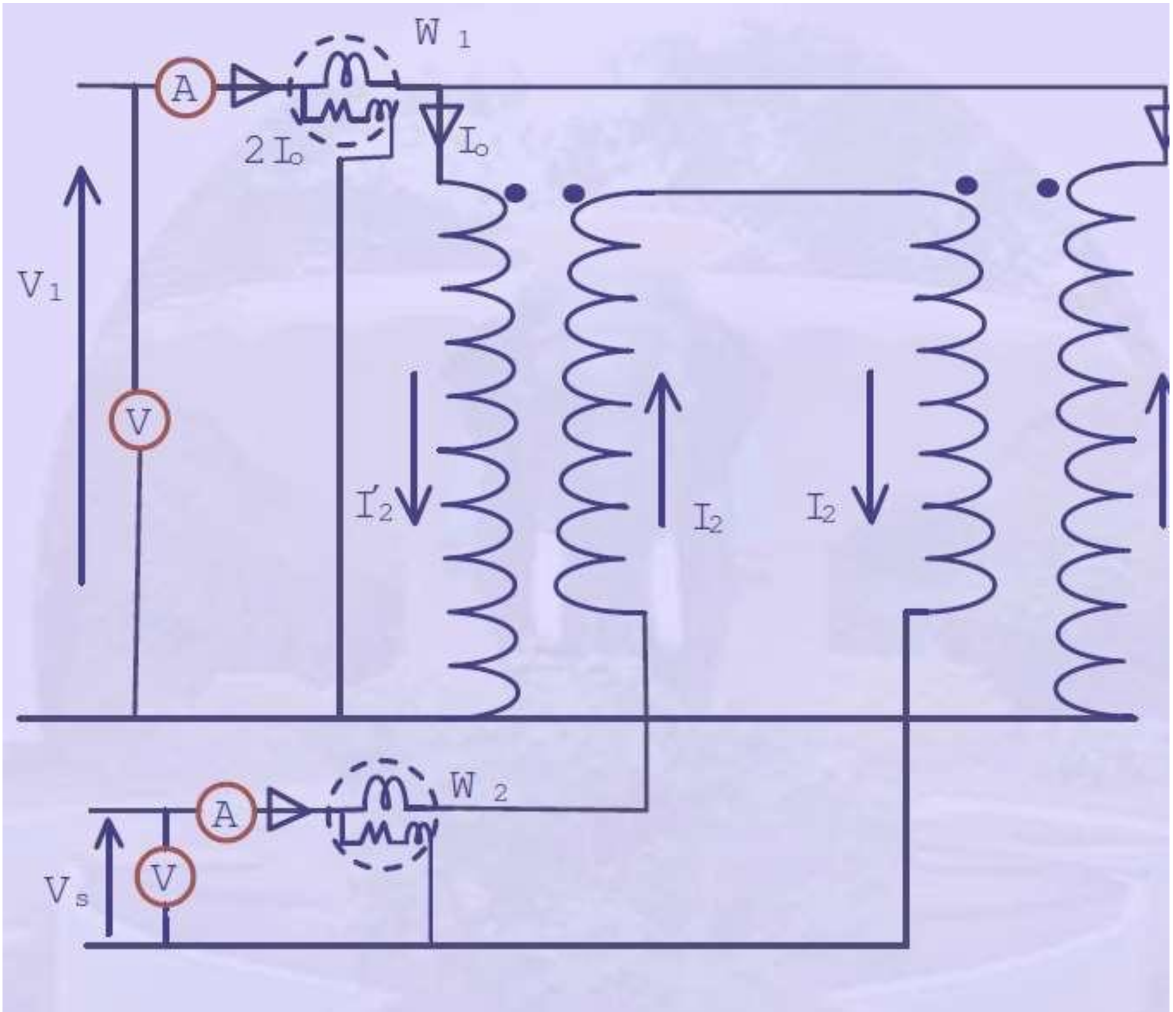


Figure 22: Back to Back Test - Phantom Loading

In Phantom loading method two identical transformers are needed. The windings are connected back to back as shown in Fig. 22. Suitable voltage is injected into the loop formed by the two secondaries such that full load current passes through them. An equivalent current then passes through the primary also. The voltage source VI supplies the magnetizing current and core losses for the two transformers. The second source supplies the load component of the current and losses due to the same. There is no power wasted in a load (as a matter of fact there is no real load at all) and hence the name Phantom or virtual loading. The power absorbed by the second transformer which acts as a load is pushed back in to the mains. The two sources put together meet the core and copper losses of the two transformers. The transformers work with full flux drawing full load currents and hence are closest to the actual loading condition with a physical load.

Voltage Regulation

Modem power systems operate at some standard voltages. The equipments working on these systems are therefore given input voltages at these standard values, within certain agreed tolerance limits. In many applications this voltage itself may not be good enough for obtaining the best operating condition for the loads. A transformer is interposed in between the load and the supply terminals in such cases. There are additional drops inside the transformer due to the load currents. While input voltage is the responsibility of the supply provider, the voltage at the load is the one which the user has to worry about.

If undue voltage drop is permitted to occur inside the transformer the load voltage becomes too low and affects its performance. It is therefore necessary to quantify the drop that takes place inside a transformer when certain load current, at any power factor, is drawn from its output leads. This drop is termed as the voltage regulation and is expressed as a ratio of the terminal voltage (the absolute value per se is not too important).

The voltage regulation can be defined in two ways - Regulation Down and Regulation up. These two definitions differ only in the reference voltage as can be seen below. Regulation down: This is defined as " the change in terminal voltage when a load current at any power factor is applied, expressed as a fraction of the no-load terminal voltage".

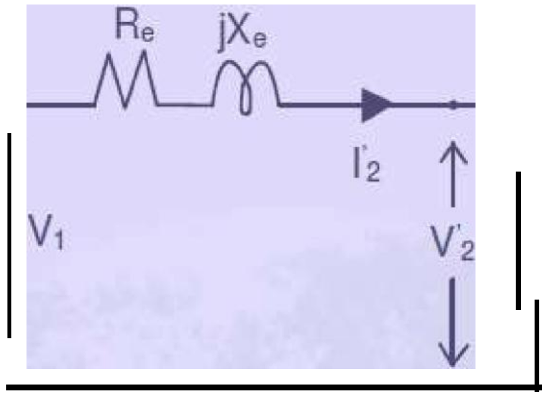
Expressed in symbolic form we have,

$$Regulation = \frac{|V_{nl}| - |V_l|}{|V_l|}$$

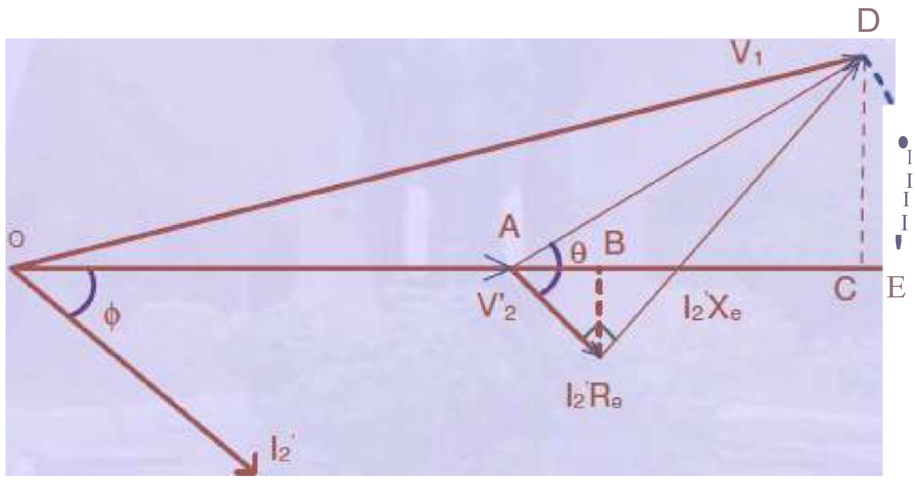
Vnl is the no-load terminal voltage. VI is load voltage. Normally full load regulation is of interest as the part load regulation is going to be lower.

This definition is more commonly used in the case of alternators and power systems as the user-end voltage is guaranteed by the power supply provider. He has to generate proper no-load voltage at the generating station to provide the user the voltage he has asked for. In the expressions for the regulation, only the numerical differences of the voltages are taken and not vector differences.

In the case of transformers both definitions result in more or less the same value for the regulation as the transformer impedance is very low and the power factor of operation is quite high. The power factor of the load is defined with respect to the terminal voltage on load. Hence a convenient starting point is the load voltage. Also the full load output voltage is taken from the name plate. Hence regulation up has some advantage when it comes to its application. Fig. 23 shows the phasor diagram of operation of the transformer under loaded condition. The no-load current IO is neglected in view of the large magnitude of I'2. Then



(a) Equi,1alent Circuit



[b)Pha or Diacram

Figur 2:3.: Regulation ,I Transformer

$I_1 = I_2'$.

½ (Re+j s)+v;'

OD 'Vi_=J[OA+ B+BC]2+[D]

$$\ldots_2' + I_2' R_e \cos\phi + I_2' X_e \sin\phi]^2 + [I_2' X_e \cos\phi - I_2' R_e \sin\phi]^2$$

,p - power factor angle,

0- internal impedance an·gle=tan-1 t

Also

$$V_1 = V_2' + I_2'.(R_e + jX_e)$$

$$= V_2' + I_2'(\cos\phi - j\sin\phi)(R_e + jX_e)$$

.•. IlegulationR $= \dfrac{|V_1| - |V_2'|}{|V_2'|} = \sqrt{(1+v_1)^2 + v_2^2} - 1$

(l+v1- +vi <) ., " l+v1)-+vi•21+v1 ,,21+vi +I2(1+v1)J $(1 + v_1 + \dfrac{v_2^2}{2(1+v_1)})$

Talci.ng h qmue root

$\sqrt{(1+v_1)^2}$ 2 2 $1 + v_1 + \dfrac{v_2^2}{2(1+v_1)}$

where "L'1 = e,. cos <p + Ber sin ¢ and v2 = e.:r cos¢ - r sin q,

= "'7T 2 per um L re J.Stanoe drop

Ex = r:,x.. =per umt reac auoe UJ·op

as V1 aJ1d 2 are mall.

_._R :::'.1 +vi+ 2(1 2 el $-1 \simeq v_1 + \dfrac{v_2^2}{2}$

.•. rel'"Ula ion R. = er cos</>± SIII'f l+ (1o:1:sin<p-er ¢)2 2

$$\frac{v_2^2}{2(1+v_1)} \simeq \frac{v_2^2}{2}.\frac{(1-v_1)}{(1-v_1^2)} \simeq \frac{v_2^2}{2}.(1-v_1) \simeq \frac{v_2^2}{2}$$

Powers higher than 2 for v1 and v2 are negligible as v**1** and v2 are already small. As v2 is small its second power may be neglected as a further approximation and the expression for the regulation of the transform boils down to regulation $R = e_r \cos\phi \pm e_x \sin\phi$

The negative sign is applicable when the power factor is leading. It can be seen from the above expression, the full load regulation becomes zero when the power factor is leading

$$e_r \cos\phi = e_x \sin\phi \text{ or } \tan\phi = e_r/e_x$$

or the power factor angle $\phi = \tan^{-1}(e_r/e_x) = \tan^{-1}(R_e/X_e)$ leading.

Similarly, the value of the regulation is maximum at a power factor angle $\phi = \tan^{-1}(e_x/e_r) = \tan^{-1}(X_e/R_e)$ lagging.

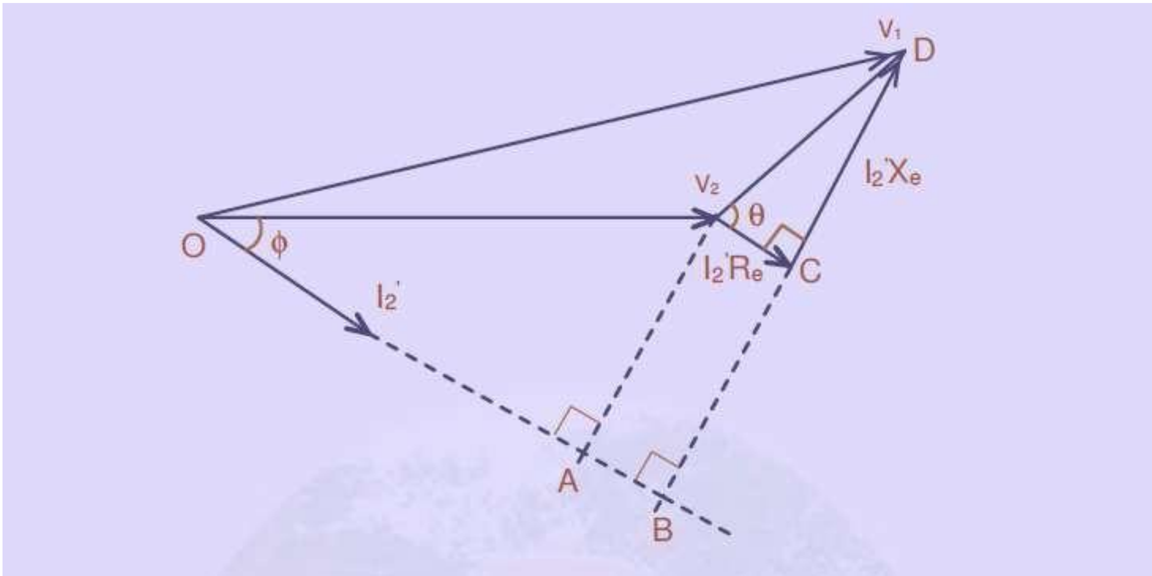
An alternative expression for the regulation of a transformer can be derived by the method shown in Fig. 24. Here the phasor are resolved along the current axis and normal to it.

We have,

$$\begin{aligned} OD^2 &= (OA+AB)^2 + (BC+CD)^2 \\ &= (V_2' \cos\phi + I_2' R_e)^2 + (V_2' \sin\phi + I_2' X_e)^2 \\ \therefore Regulation R &= \frac{OD - V_2'}{V_2'} = \frac{OD}{V_2'} - 1 \end{aligned}$$

$$\sqrt{\frac{(V_2' \cos\phi + I_2' R_e)^2}{V_2'} + \frac{(V_2' \sin\phi + I_2' X_e)^2}{V_2'}} - 1$$

$$= \sqrt{(\cos\phi + R_{p.u})^2 + (\sin\phi + X_{p.u)}^2} - 1$$



Thus this expression may not be as convenient as the earlier one due to the square root involved. Fig. shows the variation of full load regulation of a typical transformer as the power factor is varied from zero power factor leading, through unity power factor, to zero power factor lagging.

It is seen from Fig. that the full load regulation at unity power factor is nothing but the percentage resistance of the transformer. It is therefore very small and negligible. Only with low power factor loads the drop in the series impedance of the transformer contributes substantially to the regulation. In small transformers the designer tends to keep the Xe very low (less than 5%) so that the regulation performance of the transformer is satisfactory.

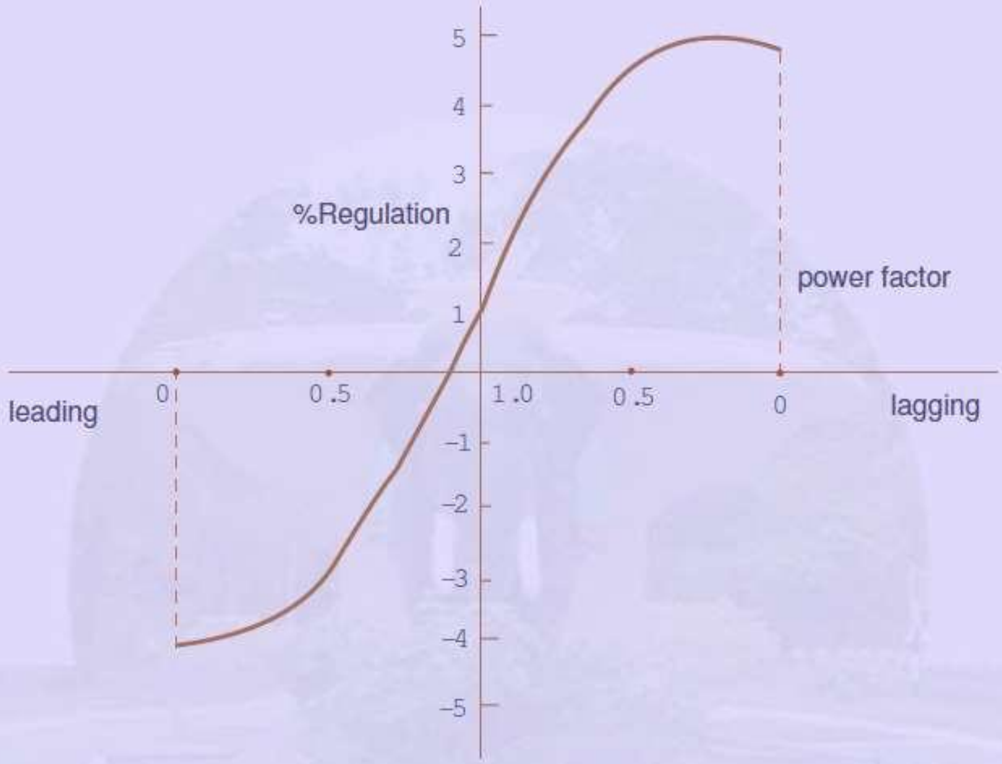


Figure 25: Variation of Full Load Regulation with Power Factor

A low value of the short circuit impedance /reactance results in a large short circuit current in case of a short circuit. This in tum results in large mechanical forces on the winding. So, in large transformers the short circuit impedance is made high to give better short circuit protection to the transformer which results in poorer regulation performance. In the case of transformers provided with taps on windings, so that the turns ratio can be changed, the voltage regulation is not a serious issue. In other cases care has to be exercised in the selection of the short circuit impedance as it affects the voltage regulation.

Efficiency

Transformers which are connected to the power supplies and loads and are in operation are required to handle load current and power as per the requirements of the load. An unloaded transformer draws only the magnetization current on the primary side, the secondary current being zero. As the load is increased the primary and secondary currents increase as per the load requirements. The volt amperes and wattage handled by the transformer also increases. Due to the presence of no load losses and I2R losses in the windings certain amount of electrical energy gets dissipated as heat inside the transformer. This gives rise to the concept of efficiency.

Efficiency of a power equipment is defined at any load as the ratio of the power output to the power input. Putting in the form of an expression,

$$\begin{aligned} Efficiency\ \eta &= \frac{output\ power}{input\ power} = \frac{Input\ power - losses\ inside\ the\ machine}{Input\ power} \\ &= 1 - \frac{losses\ inside\ the\ machine}{input\ power} = 1 - defficiency \\ &= \frac{output\ power}{output + losses\ inside\ the\ machine} \end{aligned}$$

More conveniently the efficiency is expressed in percentage. $\%\eta = \frac{output\ power}{input\ power} * 100$

While the efficiency tells us the fraction of the input power delivered to the load, the deficiency focuses our attention on losses taking place inside transformer. As a matter of fact the losses heat up machine. The temperature rise decides the rating of the equipment.

The temperature rise of the machine is a function of heat generated the structural configuration, method of cooling and type of loading (or duty cycle of load). The peak temperature attained directly affects the life of the insulations of the machine for any class of insulation.

These aspects are briefly mentioned under section 7.5 on load test.

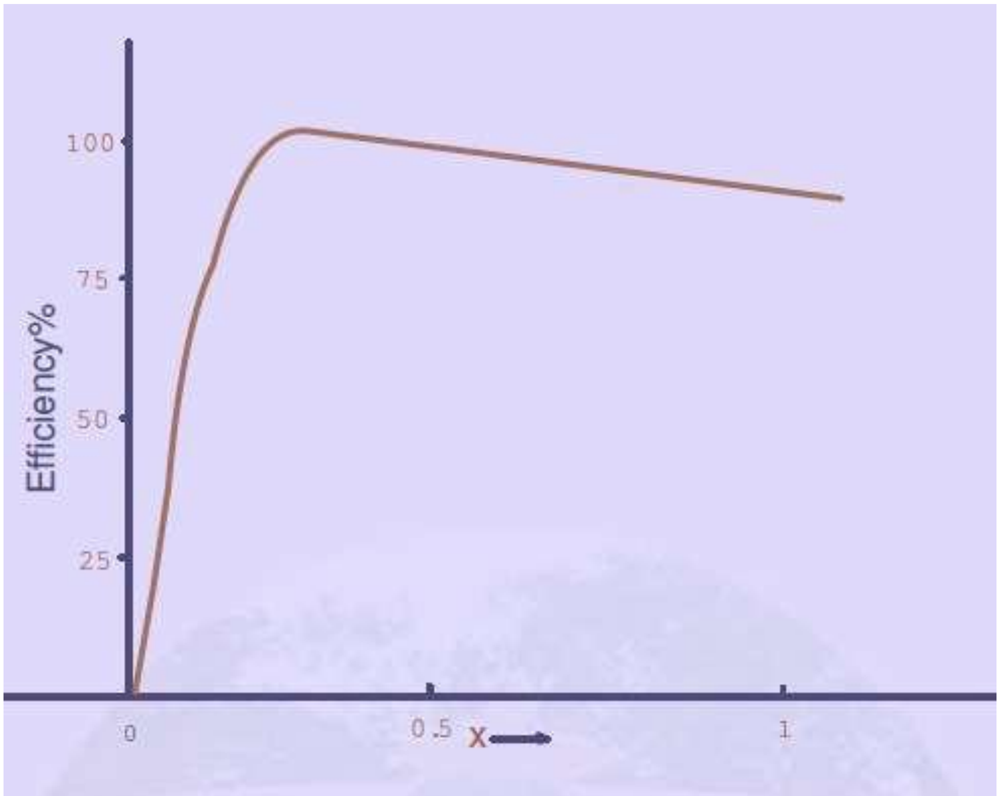


Figure 26: Efficiency

A typical curve for the variation of efficiency as a function of output is given in Fig. The losses that take place inside the machine expressed as a fraction of the input is some times termed as deficiency. Except in the case of an ideal machine, a certain fraction of the input power gets lost inside the machine while handling the power. Thus the value for the efficiency is always less than one. In the case of a.c. machines the rating is expressed in terms of apparent power. It is nothing but the product of the applied voltage and the current drawn. The actual power delivered is a function of the power factor at which this current is drawn. As the reactive power shuttles between the source and the load and has a zero average value over a cycle of the supply wave it does not have any direct effect on the efficiency. The reactive power however increases the current handled by the machine and the losses resulting from it. Therefore the losses that take place inside a transformer at any given load play a vital role in determining the efficiency. The losses taking place inside a transformer can be enumerated as below:

1. Primary copper loss
2. Secondary copper loss
3. Iron loss
4. Dielectric loss
5. Stray load loss

These are explained in sequence below.

Primary and secondary copper losses take place in the respective winding resistances due to the flow of the current in them.

$$P_c = I_1^2 r_1 + I_2^2 r_2 = I_2'^2 R_e$$

The primary and secondary resistances differ from their d.c. values due to skin effect and the temperature rise of the windings. While the average temperature rise can be approximately used, the skin effect is harder to get analytically. The short circuit test gives the value of Re taking into account the skin effect.

The iron losses contain two components - Hysteresis loss and Eddy current loss. The Hysteresis loss is a function of the material used for the core.

$$P_h = K_h B^{1.6} f$$

For constant voltage and constant frequency operation this can be taken to be constant. The eddy current loss in the core arises because of the induced emf in the steel lamination sheets and the eddies of current formed due to it. This again produces a power loss Pe in the lamination.

$$P_e = K_e B^2 f^2 t^2$$

where t is the thickness of the steel lamination used. As the lamination thickness is much smaller than the depth of penetration of the field, the eddy current loss can be reduced by reducing the thickness of the lamination. Present day laminations are of 0.25 mm thickness and are capable of operation at 2 Tesla. These reduce the eddy current losses in the core. This loss also remains constant due to constant voltage and frequency of operation. The sum of hysteresis and eddy current losses can be obtained by the open circuit test.

The dielectric losses take place in the insulation of the transformer due to the large electric stress. In the case of low voltage transformers this can be neglected. For constant voltage operation this can be assumed to be a constant.

The stray load losses arise out of the leakage fluxes of the transformer. These leakage fluxes link the metallic structural parts, tank etc. and produce eddy current losses in them. Thus they take place 'all round' the transformer instead of a definite place, hence the name 'stray'. Also the leakage flux is directly proportional to the load current unlike the mutual flux which is proportional to the applied voltage. Hence this loss is called 'stray load' loss. This can also be estimated experimentally. It can be modeled by another resistance in the series branch in the equivalent circuit. The stray load losses are very low in air-cored transformers due to the absence of the metallic tank.

Thus, the different losses fall in to two categories Constant losses (mainly voltage dependant) and Variable losses (current dependant). The expression for the efficiency of the transformer operating at a fractional load x of its rating, at a load power factor of 02, can be written as

$$\eta = \frac{xS\cos\theta_2}{xS\cos\theta_2 + P_{const} + x^2 P_{var}}$$

Here S in the volt ampere rating of the transformer (V'2 l'2 at full load), Pconst being constant losses and Pvar the variable losses at full load.

For a given power factor an expression for 02 in terms of the variable x is thus obtained. By differentiating 02with respect to x and equating the same to zero, the condition for maximum efficiency is obtained. In the present case that condition comes out to be

$$P_{const} = x^2 P_{var} \text{ or } x = \sqrt{\frac{P_{const}}{P_{var}}}$$

That is, when constant losses equal the variable losses at any fractional load x the efficiency reaches a maximum value. The maximum value of that efficiency at any given power factor is given by,

$$\eta_{max} = \frac{xS\cos\theta_2}{xS\cos\theta_2 + 2P_{const}} = \frac{xS\cos\theta_2}{xS\cos\theta_2 + 2x^2P_{var}}$$

From the expression for the maximum efficiency it can be easily deduced that this maximum value increases with increase in power factor and is zero at zero power factor of the load. It may be considered a good practice to select the operating load point to be at the maximum efficiency point. Thus if a transformer is on full load, for most part of the time then the 02max can be made to occur at full load by proper selection of constant and variable losses. However, in the modem transformers the iron losses are so low that it is practically impossible to reduce the full load copper losses to that value. Such a design wastes lot of copper.

All day efficiency

Large capacity transformers used in power systems are classified broadly into Power transformers and Distribution transformers. The former variety is seen in generating stations and large substations. Distribution transformers are seen at the distribution substations. The basic difference between the two types arise from the fact that the power transformers are switched in or out of the circuit depending upon the load to be handled by them. Thus at 50% load on the station only 50% of the transformers need to be connected in the circuit.

On the other hand a distribution transformer is never switched off. It has to remain in the circuit irrespective of the load connected. In such cases the constant loss of the transformer continues to be dissipated. Hence the concept of energy based efficiency is defined for such

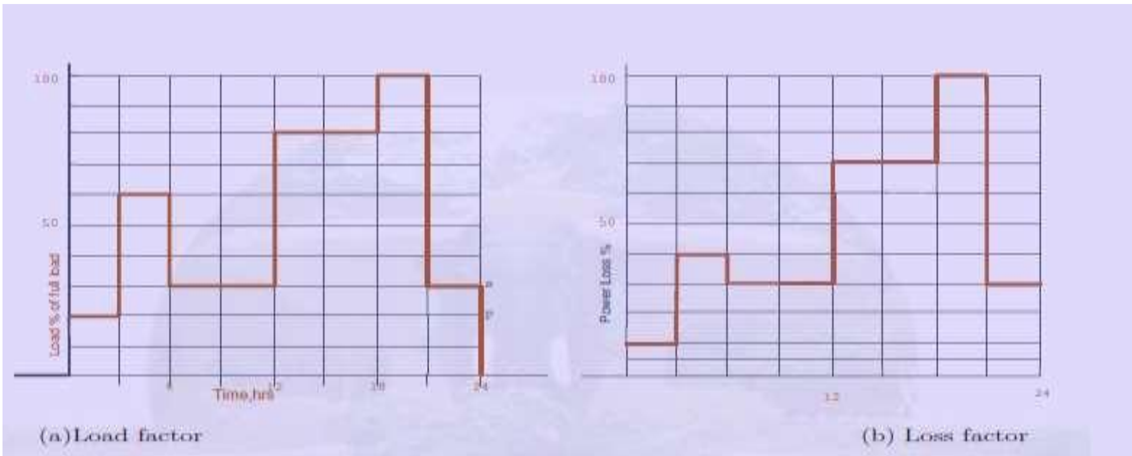


Figure 27: Calculation of Load Factor and Loss Factor

transformers. It is called 'all day' efficiency. The all day efficiency is thus the ratio of the energy output of the transformer over a day to the corresponding energy input. One day is taken as a duration of time over which the load pattern repeats itself. This assumption, however, is far from being true. The power output varies from zero to full load depending on the requirement of the user and the load losses vary as the square of the fractional loads.

The no-load losses or constant losses occur throughout the 24 hours. Thus, the comparison of loads on different days becomes difficult. Even the load factor, which is given by the ratio of the average load to rated load, does not give satisfactory results. The calculation of the all day efficiency is illustrated below with an example. The graph of load on the transformer, expressed as a fraction of the full load is plotted against time in Fig. 27. In an actual situation the load on the transformer continuously changes. This has been presented by a stepped curve for convenience. The average load can be calculated by

$$\text{Average load over a day} = \frac{\sum_{i=1}^{n} P_i}{24} = \frac{S_n \sum_{i=1}^{n} x_i t_i \cos\theta_i}{24}$$

Where Pi is the load during an interval i. n intervals are assumed. xi is the fractional load.

Si= xiSn

where Sn is nominal load. The average loss during the day is given by

$$\text{Average loss} = P_i + \frac{P_c \sum_{i=1}^{n} x_i^2 t_i}{24}$$

This is a non-linear function. For the same load factor different average loss can be there depending upon the values of xi and ti. Hence a better option would be to keep the constant losses very low to keep the all day efficiency high. Variable losses are related to load and are associated with revenue earned. The constant losses on the other hand has to be incurred to make the service available. The concept of all day efficiency may therefore be more useful for comparing two transformers subjected to the same load cycle.

The concept of minimizing the lost energy comes into effect right from the time of procurement of the transformer. The constant losses and variable losses are capitalized and added to the material cost of the transformer in order to select the most competitive one which gives minimum cost taking initial cost and running cost put together. Obviously the iron losses are capitalized more in the process to give an effect to the maximization of energy efficiency. If the load cycle is known at this stage, it can also be incorporated in computation of the best transformer.

Harmonics

In addition to the operation of transformers on the sinusoidal supplies, the harmonic behavior becomes important as the size and rating of the transformer increases. The effects of the harmonic currents are

1. Additional copper losses due to harmonic currents

2. Increased core losses

3. Increased electro magnetic interference with communication circuits.

On the other hand the harmonic voltages of the transformer cause

1. Increased dielectric stress on insulation

2. Electro static interference with communication circuits.

3. Resonance between winding reactance and feeder capacitance.

In the present times a greater awareness is generated by the problems of harmonic voltages and currents produced by non-linear loads like the power electronic converters.

These combine with non-linear nature of transformer core and produce severe distortions in voltages and currents and increase the power loss. Thus the study of harmonics is of great practical significance in the operation of transformers. The discussion here is confined to the harmonics generated by transformers only.

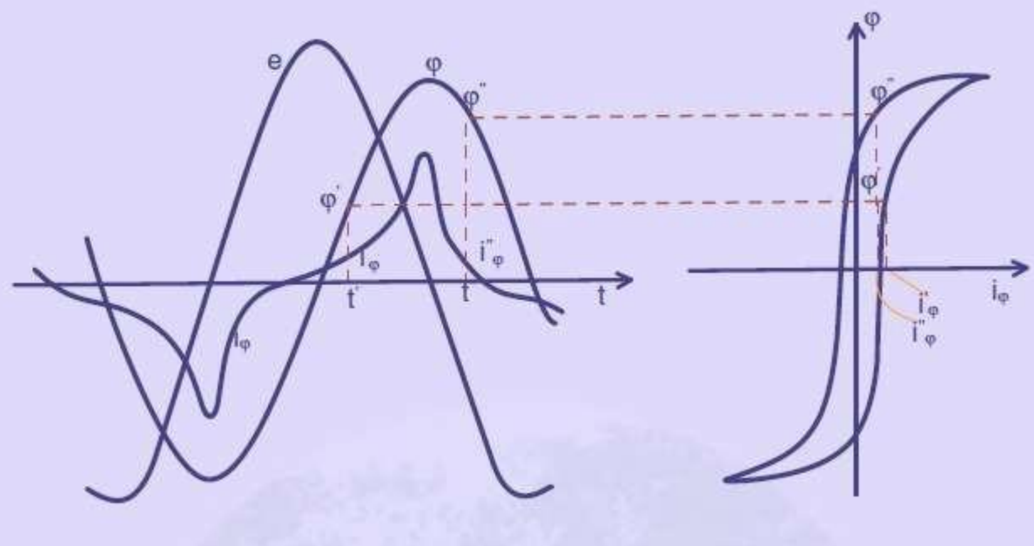


Figure 34: Harmonics Generated by Transformers

Single phase transformers

Modem transformers operate at increasing levels of saturation in order to reduce the weight and cost of the core used in the same. Because of this and due to the hysteresis, the transformer core behaves as a highly non-linear element and generates harmonic voltages and currents. This is explained below. Fig. 34 shows the manner in which the shape of the magnetizing current can be obtained and plotted. At any instant of the flux density wave the ampere turns required to establish the same is read out and plotted, traversing the hysteresis loop once per cycle. The sinusoidal flux density curve represents the sinusoidal applied voltage to some other scale. The plot of the magnetizing current which is peaky is analyzed using Fourier analysis. The harmonic current components are obtained from this analysis. These harmonic currents produce harmonic fields in the core and harmonic voltages in the windings. Relatively small value of harmonic fields generates considerable magnitude of harmonic voltages. For example a 10% magnitude of 3rd harmonic flux produces 30% magnitude of 3rd harmonic voltage. These effects get even more pronounced for higher order harmonics. As these harmonic voltages get short circuited through the low impedance of the supply they produce harmonic currents. These currents produce effects according to Lenz's law and tend to neutralize the harmonic flux and bring the flux wave to a sinusoid. Normally third harmonic is the largest in its magnitude and hence the discussion is based on it. The same can be told of other hannonics also. In the case of a single phase transformer the harmonics are confined mostly to the primary side as the source impedance is much smaller compared to the load impedance. The understanding of the phenomenon becomes more clear if the transformer is supplied with a sinusoidal current source. In this case current has to be sinusoidal and the harmonic currents cannot be supplied by the source and hence the induced emf will be peaky containing harmonic voltages. When the load is connected on the secondary side the harmonic currents flow through the load and voltage tends to become sinusoidal. The harmonic voltages induce electric stress on dielectrics and increased electro static interference. The harmonic currents produce losses and electro magnetic interference as already noted above.

Poly Phase connections and Poly phase Transformers

The individual transformers are connected in a variety of ways in a power system. Due to the advantages of polyphase power during generation, transmission and utilization polyphase power handling is very important. As an engineering application is driven by techno-economic considerations, no single connection or setup is satisfactory for all applications. Thus transformers are deployed in many forms and connections. Star and mesh connections are very commonly used. Apart from these, vee or open delta connections, zigzag connections , T connections, auto transformer connections, multi winding transformers etc. are a few of the many possibilities. A few of the common connections and the technical and economic considerations that govern their usage are discussed here. Literature abounds in the description of many other. Apart from the characteristics and advantages of these, one must also know their limitations and problems, to facilitate proper selection of a configuration for an application.

Many polyphase connections can be formed using single phase transformers. In some cases it may be preferable to design, develop and deploy a polyphase transformer itself. In a balanced two phase system we encounter two voltages that are equal in magnitude differing in phase by 90°. Similarly, in a three phase system there are three equal voltages differing in phase 120 electrical degrees. Further there is an order in which they reach a particular voltage magnitude. This is called the phase sequence. In some applications like a.c. to d.c. conversion, six phases or more may be encountered. Transformers used in all these applications must be connected properly for proper functioning. The basic relationship between the primary and secondary voltages (brought about by a common mutual flux and the number of turns), the polarity of the induced emf (decided by polarity test and used with dot convention) and some understanding of the magnetic circuit are all necessary for the same. To facilitate the manufacturer and users, international standards are also available. Each winding has two ends designated as 1 and 2. The HV winding is indicated by capital letters and the LV winding by small letters. If more terminals are brought out from a winding by way of taps there are numbered in the increasing numbers in accordance to their distance from 1 (eg Al,A2,A3...). If the induced emf at an instant is from A1 to A2 on the HV winding it will rise from al to a2 on the LV winding.

Out of the different polyphase connections three phase connections are mostly encountered due to the wide spread use of three phase systems for generation, transmission and utilization. Three balanced 3-phase voltages can be connected in star or mesh fashion to yield a balanced 3-phase 3-wire system. The transformers that work on the 3-phase supply have star, mesh or zig-zag connected windings on either primary secondary or both. In addition to giving different voltage ratios, they introduce phase shifts between input and output sides. These connections are broadly classified into 4 popular vector groups.

1. Group I: zero phase displacement between the primary and the secondary.

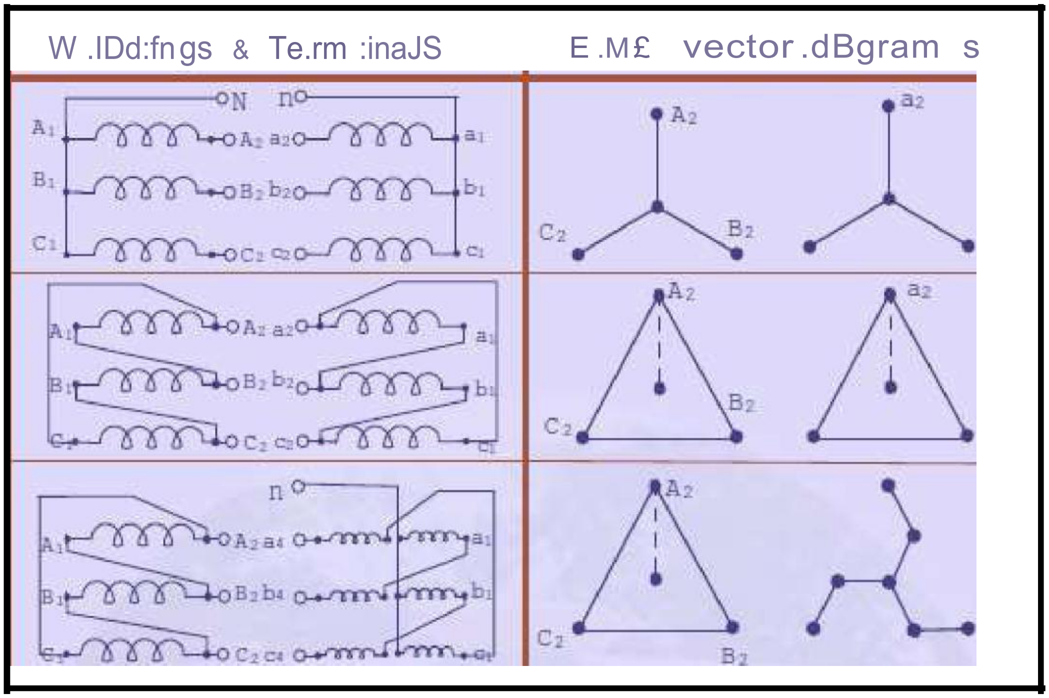
2. Group II: 180° phase displacement.

3. Group III: 30° lag phase displacement of the secondary with respect to the primary.

4. Group IV: 30° lead phase displacement of the secondary with respect to the primary.

A few examples of the physical connections and phasor diagrams are shown in Fig. 35 and Fig. 36 corresponding to each group. The capital letters indicates primary and the small letters the secondary. *Did* stand for mesh, Y/y - for star, Z/z for zig-zag. The angular displacement of secondary with respect to the primary are shown as clock position, O^0

Group1 □0 _base shm.



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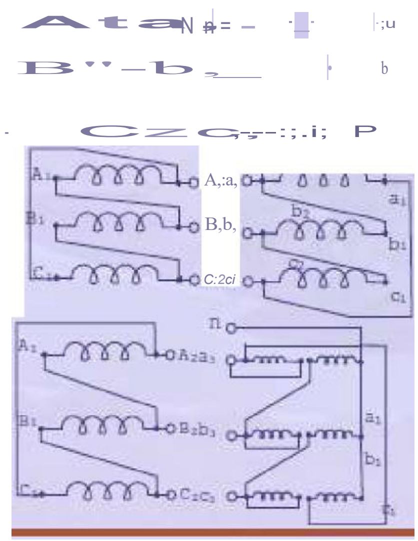	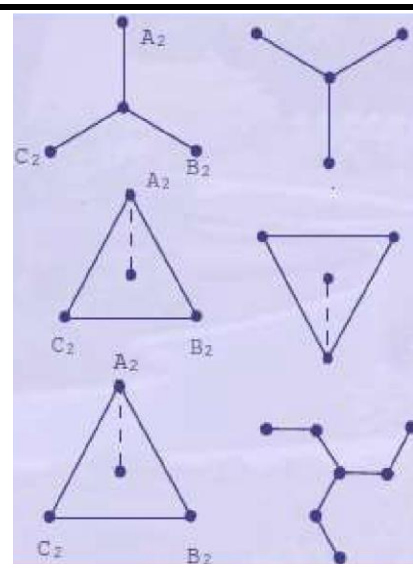

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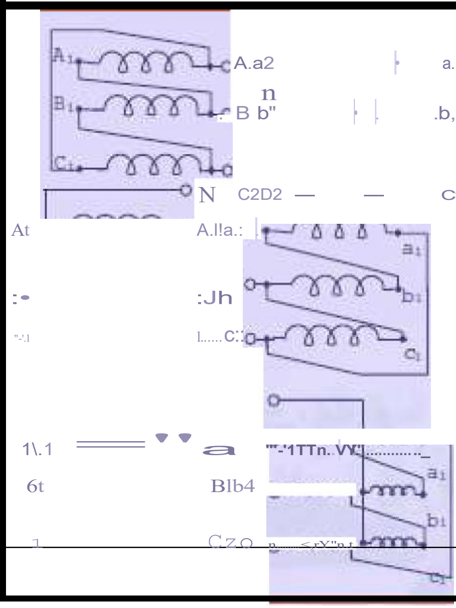
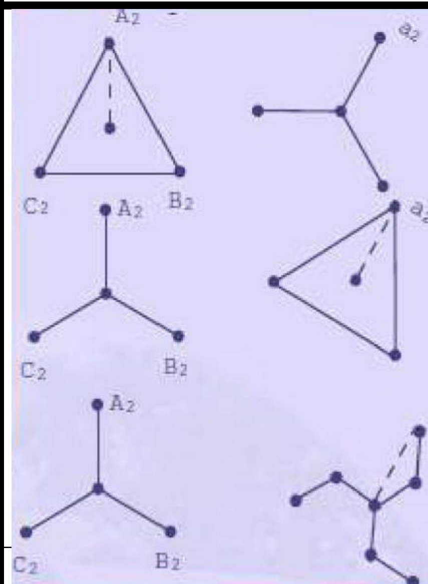
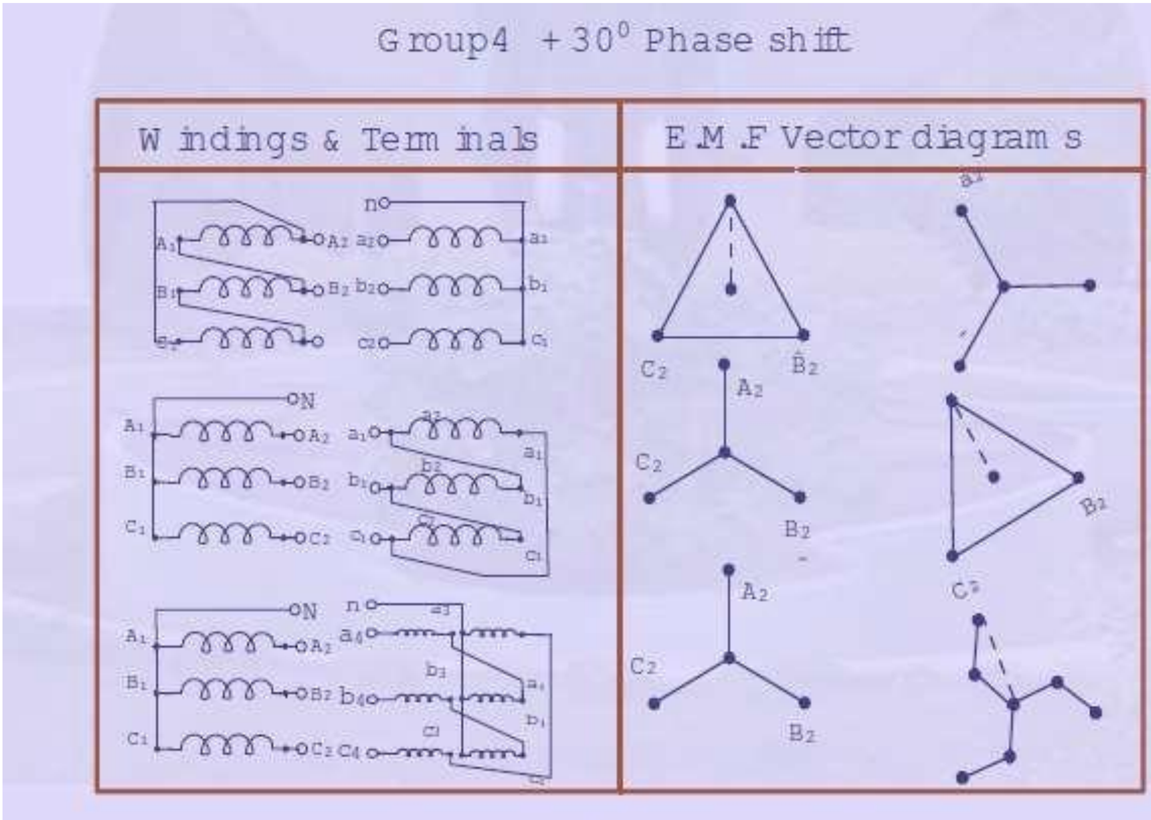
W _nd_:_gs & Tenn na	E .M .F vectordI:I.gram s
	



(b)

referring to 12 o'clock position. These vector groups are especially important when two or more transformers are to be connected in parallel.

Star connection is normally cheaper as there are fewer turns and lesser cost of insulation. The advantage becomes more with increase in voltage above 1 lkv. In a star connected winding with earthed-neutral the maximum voltage to the earth is (13)of the line voltage.

Also star connection permits mixed loading due to the presence of the neutral. Mesh connections are advantageous in low voltage transformers as insulation costs are insignificant and the conductor size becomes (13) of that of star connection and permits ease of winding. The common polyphase connections are briefly discussed now.

Star/star (Yy0, Yy6) connection This is the most economical one for small high voltage transformers. Insulation cost is highly reduced. Neutral wire can permit mixed loading. Triplen harmonics are absent in the lines. These triplen harmonic currents cannot flow, unless there is a neutral wire. This connection produces oscillating neutral. Three phase shell type units have large triplen harmonic phase voltage. However three phase core type transformers work satisfactorily. A tertiary mesh connected winding may be required to stabilize the oscillating neutral due to third harmonics in three phase banks.

Mesh/mesh (Dd0, Dd6) This is an economical configuration for large low voltage transformers. Large amount of unbalanced load can be met with ease. Mesh permits a circulating path for triplen harmonics thus attenuates the same. It is possible to operate with one transformer removed in open delta or Vee connection meeting 58 percent of the balanced load. Three phase units cannot have this facility. Mixed single phase loading is not possible due to the absence of neutral.

Star/mesh(Dy or Yd) This arrangement 1s very common for power supply transformers. The delta winding permits triplen harmonic currents to circulate in the closed path and attenuates them.

Zig zag/ star (ZYI or Zyll) Zigzag connection is obtained by inter connection of phases. 4-wire system is possible on both sides. Unbalanced loading is also possible. Oscillating neutral problem is absent in this connection. This connection requires 15% more turns for the same voltage on the zigzag side and hence costs more.

Generally speaking a bank of three single phase transformers cost about 15% more than their 3-phase counter part. Also, they occupy more space. But the spare capacity cost will be less and single phase units are easier to transport.

Mesh connected three phase transformers resemble 3- single phase units but kept in a common tank. In view of this single tank, the space occupied is less. Other than that there is no big difference. The 3-phase core type transformer on the other hand has a simple core arrangement. The three limbs are equal in cross section. Primary and secondary of each phase are housed on the same limb. The flux setup in any limb will return through the other two limbs as the mmf of those limbs are in the directions so as to aid the same. Even though magnetically this is not a symmetrical arrangement, as the reluctance to the flux setup by side limbs is different from that of the central limb, it does not adversely affect the performance. This is due to the fact that the magnetizing current itself forms a small fraction of the total phase current drawn on load. The added advantage of 3-phase core is that it can tolerate substantially large value of 3rd harmonic mmf without affecting the performance. The 3rd harmonic mmf of the three phases will be in phase and hence rise in all the limbs together.

The 3rd harmonic flux must therefore find its path through the air. Due to the high reluctance of the air path even a substantially large value of third harmonic mmf produces negligible value of third harmonic flux. Similarly unbalanced operation of the transformer with large zero sequence fundamental mmf content also does not affect its performance. Even with Yy type of poly phase connection without neutral connection the oscillating neutral does not occur with these cores. Finally, three phase cores themselves cost less than three single phase units due to compactness.