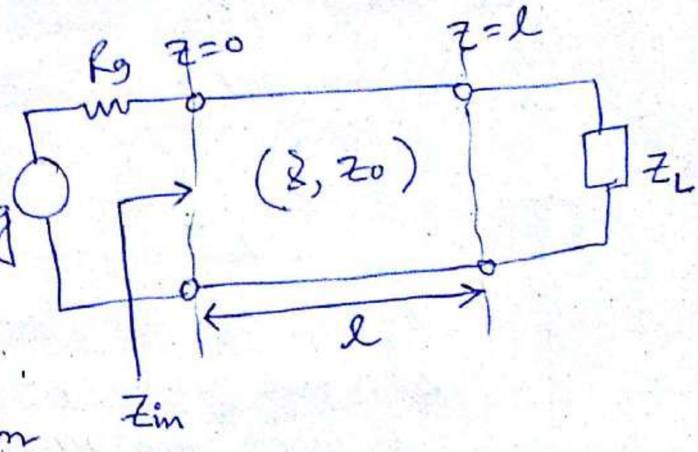
Input Impedance

Input impedance is the line impedance seen at the beging of the transmission line. The i/p impedance at any point of a transmission line is given by the ratio of the i/p voltage wave to the current wave at that point.



Consider the transmission line of length l, characterized by propagation constant γ and characteric impedance Z_0, its connected to the load as shown in figure. Let, the transmission line extend from $z=0$ at generator or sending end to $z=l$ at load end. We have the general voltage and current wave equations in the form of

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \text{---} (1)$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z} \quad \text{---} (2)$$

From the defination of characteristics impedance,

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-}$$

$$I_0^+ = \frac{V_0^+}{Z_0} \quad \text{and} \quad I_0^- = -\frac{V_0^-}{Z_0}$$

$$\therefore I_s(z) = \frac{1}{Z_0}\left[V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right] \quad \text{---} (3)$$

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At the sending end, i.e., $z=0$, the voltage and current wave eq^{n}s. are reduce to

$$V_s(z=0) = V_s = V_0^+ + V_0^- \quad \text{---} (4)$$

$$I_s(z=0) = I_s = \frac{1}{Z_0}\left(V_0^+ - V_0^-\right) \quad \text{---} (5)$$

from eq^{n}s (4) & (5), we can findout,

$$V_0^+ = \frac{V_s + Z_0 I_s}{2} \quad \text{----} (6)$$

$$V_0^- = \frac{V_s - Z_0 I_s}{2} \quad \text{---} (7)$$

So, the voltage and current wave eq^n. at any distance z are express as

$$V_s(z) = \frac{V_s + Z_0 I_s}{2} e^{-\gamma z} + \frac{V_s - Z_0 I_s}{2} e^{\gamma z}$$

$$\text{or, } V_s(z) = \frac{1}{2} V_s \left(e^{\gamma z} + e^{-\gamma z}\right) - \frac{1}{2} Z_0 I_s \left(e^{\gamma z} - e^{-\gamma z}\right)$$

$$\text{or, } V_s(z) = V_s \cosh \gamma z - Z_0 I_s \sinh \gamma z \quad \text{-} (8)$$

Similarly,

$$I_s(z) = \frac{1}{2} I_s \left(e^{\gamma z} + e^{-\gamma z}\right) - \frac{1}{2} \frac{V_s}{Z_0} \left(e^{\gamma z} - e^{-\gamma z}\right)$$

$$\text{or, } I_s(z) = I_s \cosh \gamma z - \frac{V_0}{Z_0} \sinh \gamma z \quad \text{--} (9)$$

At the receiving end, at $z=l$, the voltage & current wave eq^{n}s (8) & (9) can be written as

$$V_s(z=l) = V_L = V_s \cosh \gamma l - Z_0 I_s \sinh \gamma l \quad \text{--} (10)$$

$$I_s(z=l) = I_L = I_s \cosh \gamma l - \frac{V_s}{Z_0} \sinh \gamma l \quad \text{--} (11)$$

$$\frac{e^{\gamma z} + e^{-\gamma z}}{2} = \cosh \gamma z \qquad \frac{e^{+\gamma z} - e^{-\gamma z}}{2} = \sinh \gamma z$$

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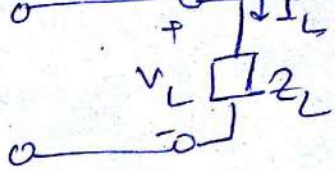
When the receiving end of a transmission line is terminated by a load impedance then the voltage across Z_L will be

$$V_L = Z_L I_L \quad \text{--- (12)}$$



Substitute the V_L & I_L in eqn (12), as.

$$V_S \cosh\gamma l - Z_0 I_S \sinh\gamma l = Z_L \cdot I_S \cosh\gamma l - \frac{Z_L V_S}{Z_0} \sinh\gamma l$$

or, $V_S \left(\cosh\gamma l + \frac{Z_L}{Z_0} \sinh\gamma l \right) = I_S \left(Z_L \cosh\gamma l + Z_0 \sinh\gamma l \right)$

at the beging of the transmission line,

$$Z_{in} = \frac{V_S}{I_S} = \frac{Z_L \cosh\gamma l + Z_0 \sinh\gamma l}{\cosh\gamma l + Z_L/Z_0 \sinh\gamma l}$$

$$= \frac{Z_L + Z_0 \tanh\gamma l}{1 + \frac{Z_L}{Z_0} \tanh\gamma l}$$

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + Z_0 \tanh\gamma l}{Z_0 + Z_L \tanh\gamma l} \quad \text{--- (13)}}$$

This is the i/p impedance for finite lossy transmission line.

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For lossless transmission line,

$$\gamma = \alpha + j\beta \text{ , where, } \alpha = 0$$

i.e., $\gamma = j\beta$

Hence, $\tan(h j\beta l) = j\tan\beta l$.

then,

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}}$$

I/p impedance for loss less transmission line.

(II) Reflection coefficient

Reflection coefficient of a transmission line is the ratio of the reflected voltage (or current) to the incident voltage (or current), when a transmission line is terminated in an impedance Z_L not equal to the characteristic impedance (Z_0) of the line.

The general transmission-line eqn. for voltage & current as a function of position along the line are,

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z} \quad \text{--- (1)}$$

and $$I_s(z) = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma z} - V_0^- e^{+\gamma z}\right) \quad \text{--- (2)}$$

The voltage and current at the load $(z = l)$ are given as

$$V_s(z=l) = V_L = V_0^+ e^{-\gamma l} + V_0^- e^{+\gamma l} \quad \text{--- (3)}$$

and $$I_s(z=l) = I_L = \frac{1}{Z_0}\left(V_0^+ e^{-\gamma l} - V_0^- e^{+\gamma l}\right) \quad \text{--- (4)}$$

Solving these two eqns., we get the voltage coefficient in terms of the load voltage and load current as follows

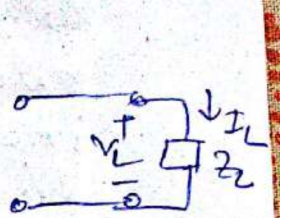
$$V_0^+ = \left(\frac{V_L + Z_0 I_L}{2}\right) e^{\gamma l} \quad \text{--- (5)}$$

$$V_0^- = \left(\frac{V_L - Z_0 I_L}{2}\right) e^{-\gamma l} \quad \text{--- (6)}$$

The voltage reflection coefficient as a function of position along the line $[\Gamma(z)]$ is defined as the ratio of the reflected voltage wave to the transmitted voltage wave.

$$\Gamma(z) = \frac{V_0^- e^{+\gamma z}}{V_0^+ e^{-\gamma z}} = \frac{V_0^-}{V_0^+} e^{+2\gamma z} \quad \text{--- (7)}$$

Inserting the expressions for the voltage coefficients in term of the load voltage and current, we have,



$$\Gamma(z) = \frac{\left(\frac{V_L - Z_0 I_L}{2}\right) e^{-\gamma l}}{\left(\frac{V_L + Z_0 I_L}{2}\right) e^{+\gamma l}} \cdot e^{2\gamma z}$$

$$= \frac{Z_L - Z_0}{Z_L + Z_0}\, e^{-2\gamma l} \cdot e^{2\gamma z}$$

$$\boxed{\therefore \Gamma(z) = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot e^{2\gamma(z-l)}}$$

The reflection coefficient at the load $(z=l)$ is

$$\boxed{\Gamma(z=l) = \Gamma_l = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

Hence, the reflection coefficient as a function of position can be written as,

$$\boxed{\Gamma(z) = \frac{Z_L - Z_0}{Z_L + Z_0} e^{2\gamma(z-l)} = \Gamma_L e^{2\gamma(z-l)}}$$

Note

⇒ The current reflection coefficient at any point on the line is the negative of the voltage reflection coefficient at that point.

(11) VSWR (voltage standing wave Ratio)

→ Generally, if $Z_L = Z_0$, then reflection coefficient, $\Gamma_L = 0$, that means, the transmission line is perfectly matched. This is the ideal case where no reflections occurs from the load and all the energy associated with the forward-travelling wave is delivered to the load.

→ If the receiving end of a transmission line is not perfectly matched, there will be reflection of the voltage and current.

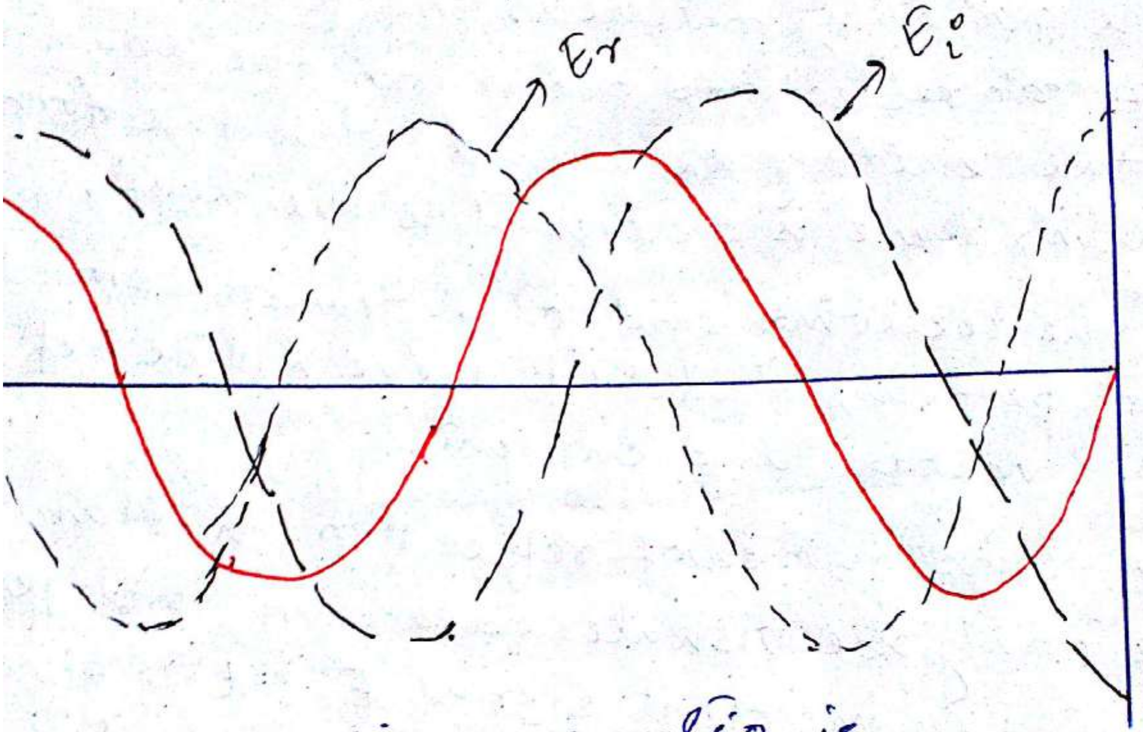
→ As a consequence of reflection, a standing wave may be visualised as an interference between the incident signal E_i at a given frequency, travelling in the forward direction, and the reflected signal E_r, at the same frequency, travelling in the reverse direction.

→ At the load, the relationship between the amplitudes of E_r and E_i and the phase angle between them are uniquely determined by the load impedance.

→ The phase angle between E_r and E_i, however, will vary along the line as a function of the distance from the load.

→ Since the wave oscillates in amplitude but never moves laterally, it is called a standing wave.

→ In fig, the incident, reflected and the Standing waves can be seen. The dashed lines are the E_r and E_i, while the nondashed one represents the standing wave.



→ The standing wave ratio is defined as the ratio of the maximum voltage (or current) to the minimum voltage (or current) of a line having standing waves

$$\boxed{S = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}}$$

(11) Establish the ~~rd~~ relationship between $\boxed{S = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}}$

⇒ We have for a lossless line, the voltage and current given as,

$$V_S(z) = V_0^+ e^{-j\beta z} + V_0^- e^{+j\beta z} \quad [\because \alpha = 0]$$
$$I_S(z) = \frac{1}{Z_0}(V_0^+ e^{-j\beta z} - V_0^- e^{+j\beta z})$$

If we let x be the distance from the receiving end, the above two equations can be expressed in terms of x simply by putting $z = -x$,

$$V_S(z=x) = V_0^+ e^{j\beta x} + V_0^- e^{-j\beta x}$$

$$I_S(x) = \frac{1}{Z_0}\left(V_0^+ e^{j\beta x} - V_0^- e^{-j\beta x}\right)$$

Now, reflection coefficient is given as,

$$\Gamma = |\Gamma| e^{j\theta} = \frac{V_0^- e^{-j\beta x}}{V_0^+ e^{+j\beta x}} = \left(\frac{V_0^-}{V_0^+}\right) e^{-2j\beta x}$$

At receiving end, $x = 0$ and

$$\Gamma = \Gamma_l = |\Gamma| e^{j\theta} = \left(\frac{V_0^-}{V_0^+}\right)$$

$$\boxed{V_0^- = V_0^+ |\Gamma| e^{j\theta}}$$

Repalacing this value, we get,

$$V_S(x) = V_0^+ e^{j\beta x} + V_0^+ |\Gamma| e^{j\theta} \cdot e^{-j\beta x}$$

$$V_s(x) = V_0^+ \left[e^{j\beta x} + |\Gamma| e^{j\theta} e^{-j\beta x} \right]$$

$$= V_0^+ e^{j\beta x} \left[1 + |\Gamma| e^{j\theta} \cdot e^{-2j\beta x} \right]$$

$$= V_0^+ e^{j\beta x} \left[1 + |\Gamma| e^{-j(2\beta x - \theta)} \right]$$

For the voltage to be maximum, two components must be in phase, so, that.

$$\left(2\beta x_{max} - \theta \right) = 2n\pi$$

where, $n = 0, 1, 2, 3, \ldots$

The first voltage maximum nearest to the load end will occur when $n = 0$, i.e., $2\beta x_{max} = \theta$.

The magnitude of the maximum voltage is given as,

$$V_{s\,max} = V_0^+ \left[1 + |\Gamma| \right]$$

For the voltage to be minimum, two components must be in phase opposition, so that,

$$\left(2\beta x_{min} - \theta \right) = (2n+1)\pi$$

where $n = 0, 1, 2, 3, \ldots$

The first voltage minimum nearest to the load end will occur when $n = 0$, i.e.,

$$\left(2\beta x_{min} - \theta \right) = \pi$$

The magnitude of the minimum voltage is given as,

$$V_{s\,min} = V_0^+ \left[1 - |\Gamma| \right]$$

If we take the ratio of maximum and minimum voltages, we get,

$$\boxed{\frac{V_{s\,max}}{V_{s\,min}} = \frac{1+|\Gamma|}{1-|\Gamma|} = VSWR(S)}$$

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$$\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}} = \frac{V_r}{V_i}$$

$V_r \to$ reflected wave

$V_i =$ incident wave.

$$= \frac{Z_L - Z_0}{Z_L + Z_0}$$

The general from of wave eqn.

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{+\gamma z}$$

at load, $(z = l)$

$$V_s(z=l) = V_0^+ e^{-\gamma l} + V_0^- e^{+\gamma l}$$

magnitude of $= V_i + V_r$.

The voltage wave is maximum, when V_i & V_r are in same phase.

$$|V_{max}| = |V_i| + |V_r|$$

The magnitude of voltage wave is minimum, when V_i & V_r are in opposite phase,

$$|V_{min}| = |V_i| - |V_r|$$

now,

$$VSWR(S) = \frac{|V_{max}|}{|V_{min}|} = \frac{|V_i| + |V_r|}{|V_i| - |V_r|}$$

$$S = \frac{1 + \left|\frac{V_r}{V_i}\right|}{1 - \left|\frac{V_r}{V_i}\right|} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}$$